The Effect of the Δ (1236)-Resonance in Nuclear Matter

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The effect of the resonant $\Delta(1236)$-states in nuclear matter, is studied within the framework of the exp(S)-formalism [1, 2]. Treating the Schrödinger-Equation of the ground state of an A-particle fermion system with the help of the exp(S)-formalism one obtains a system of A coupled equations which all together are equivalent to the A particle Schrödinger equation. Neglecting three- and more particle forces as well as more particle effects the ground state of the A-particle system is described by the coupled system of one particle and two-particle-equations. Considering the special conditions of nuclear matter the one particle equations turn out to be trivial while the two particle equations reduce to a generalized Bethe-Goldstone-equation. After decomposition of these equations into partial waves we obtain a computable set of coupled integro-differential equations. These equations are fully selfconsistently solved including all partial waves up to total spin $J = 2$. Numerical calculation shows that the effects produced by Δ-Δ-partial waves are comparable to those produced by N-Δ-partial waves. Transitions from the N-N channels to N-Δ or Δ-Δ channels are caused by nonrelativistic potentials obtained from the static limit of meson theory. The N-N-interaction is described by a Reid-potential which is modified in order to reproduce the two particle dates (N-N phase shifts, deuteron).

I. Introduction

Nuclear matter is a hypothetical system of an equal number of neutrons and protons filling the whole space with uniform density, which is fortunately not realized by nature. The Coulomb force is assumed to be turned off. It seems meaningful for the following reasons to get down to such a system.

First of all translational invariance, neglecting of Coulomb forces and the absence of a surface cause tremendous simplifications of nuclear matter calculations compared to the finite nucleus problem. As the one particle problem doesn't exist for nuclear matter — we know, the one particle wave functions to be plane waves — we have the unique opportunity to study some two particle effects much more easily as this would be possible in the case of the finite nucleus.

Next we may expect that nuclear matter calculations can be helpful in deciding which two-body force is realistic. A two-body force that does produce a very unreasonable saturation curve or doesn't cause saturation at all can be discarded. Finally we note that the fact that nuclear matter is a hypothetical system doesn't mean that nature didn't realize many-body-systems which are not describable by nuclear matter methods.

All nuclear-matter calculations done so far within the framework of Bethe-Goldstone-Theorie [3, 4, 5], show a remarkable effect. For various N-N-potentials the calculated saturation points lie on a narrow band in the energy-density plane that doesn't contain the "empirical" point

$$(\varepsilon_0, \varrho_0) = (-16\,\mathrm{MeV},\ 0.170\,\mathrm{fm}^{-3}) \quad [6].$$

Derivation of a two particle equation for nuclear matter from the exp(S)-theorie leads to a generalized Bethe-Goldstone-equation. The result of a numerical calculation based on this two body equation consists in about 0.5 MeV more binding energy at saturation density than comparable calculations based on the Bethe-Goldstone equation. Numerically this calculation is about 100 times more extensive than that done by Kallio and Day [3]. Nevertheless we still miss about 4 MeV binding energy. So we come to the conclusion that processes including only two nucleons are not sufficient if we want to obtain the correct saturation point for nuclear matter. Recently people have tried to escape the saturation band following two completely different ways.

On the one hand one gives up the restriction of solving a two-particle equation. Instead one tries to treat three and four-particle effects by solving the corresponding equations. Following this way one has to deal with two great problems. The numerical problem of solving a three- or four-particle equation is very hard.

The second problem is how to take three- and more-particle forces into consideration. Without serious approximations these forces cannot be

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calculated nor is it possible to fit them to reproduce some empirical dates.

On the other hand one goes on dealing with two-particle equations but gives up the restriction of dealing with proton and neutron alone. Instead one tries to take into account the excited nucleons the resonances explicitly. Unfortunately this way too is not without problems. One problem is to be seen in the description of the two-body-interaction. One can't calculate such potentials without using free parameters. We get some certainty however from the advantage of being able to fit our potentials in order to reproduce the two particle dates. Moreover we are confronted with an enormous numerical problem. By taking into account all partial waves with total spin up to $J = 2$ we get integro-differential systems coupling up to eight partial waves. So numerical calculations takes about twenty times longer than calculating the ordinary nuclear matter problem.

II. The exp(S)-formalism [1, 2, 7]

The $\exp(S)$-formalism gives the possibility to describe correlations of any order to the shell-model systematically without making use of diagramatic methods. By restriction to a definit order of the correlations one gets equations which are generalized Brueckner-Hartree-Fock-, generalized Bethe-Goldstone-equations etc.

We know that the ground state of a nondegenerate A-nucleon-system $|\Psi\rangle$ can be created from a shell model determinant $|\Phi\rangle$ by:

$$|\Psi\rangle = \exp(S)\,|\Phi\rangle\,, \tag{2.1}$$

$$|\Phi\rangle = \prod_{\nu=1}^{A} a_\nu{}^+\,|0\rangle\,; \quad S = \sum_{n=1}^{A} S_n\,, \tag{2.2}$$

$$S_n = \frac{1}{(n!)^2} \sum_{\substack{\varrho_1\ldots\varrho_n\\ \nu_1\ldots\nu_n}} a^+_{\varrho_n}\ldots a^+_{\varrho_1} \cdot \langle\varrho_1\ldots\varrho_n|S_n|\nu_1\ldots\nu_n\rangle_{\mathrm{A}}\, a_{\nu_1}\ldots a_{\nu_n}\,. \tag{2.3}$$

The operator S_n destroys n particles $\nu_1\ldots\nu_n$ in the Fermi sea and creates n particles $\varrho_1\ldots\varrho_n$ above the Fermi sea. This means that $\exp(S)$ develops $|\Psi\rangle$ in form of n-particle-n-hole-excitations out of $|\Phi\rangle$. The occupation probabilities for these excitations are determined by $\langle\varrho_1\ldots\varrho_n|S_n|\nu_1\ldots\nu_n\rangle$. The fact that we just use the operator $\exp(S)$ for this development is justified by a proveable linked cluster theorem concerning the S_n. This means that only connected diagrams occur if one represents the contributions to the binding energy by diagrams just as it is done in the Brueckner-Goldstone theory. This is meaningful because in this form all contributions to the binding energy are proportional to the particle number A. Demanding that $|\Psi\rangle$ is a solution of the Schrödinger-equation and normalizing $\langle\Phi|\Psi\rangle = 1$ one can derive equations for the correlation functions connected with the S_n.

$$\begin{aligned}&\langle x_1\ldots x_n|S_n|\nu_1\ldots\nu_n\rangle_{\mathrm{A}}\\ &= \sum_{\varrho_1\ldots\varrho_n} \langle x_1|\varrho_1\rangle\cdots\langle x_n|\varrho_n\rangle\\ &\quad\cdot\langle\varrho_1\ldots\varrho_n|S_n|\nu_1\ldots\nu_n\rangle_{\mathrm{A}}\,.\end{aligned} \tag{2.4}$$

It is important that we are able to choose a shell model determinant such that all contributions corresponding to S_1 vanish.

To get equations which describe the correlation functions we project the Schrödinger equation successively by:

$$\langle\Phi|\,, \tag{2.5}$$

$$\langle\Phi|\,a^+_{\nu_1}\,a(x_1)\,, \tag{2.6}$$

$$\langle\Phi|\,a^+_{\nu_1}\,a^+_{\nu_2}\,a(x_2)\,a(x_1) \quad \text{etc.} \tag{2.7}$$

This leads to a system of A coupled equations which alltogether are equivalent to the Schrödinger equation. Particularly, we obtain:

$$\langle\Phi|H\,e^s|\Phi\rangle = E\,, \tag{2.8}$$

$$\begin{aligned}&\langle\Phi|\,a^+_{\nu_1}\,a(x_1)\,H\,e^s|\Phi\rangle\\ &\quad-\langle\Phi|H\,e^s|\Phi\rangle\langle\Phi|\,a^+_{\nu_1}\,a(x_1)\,e^s|\Phi\rangle = 0\,,\end{aligned} \tag{2.9}$$

$$\begin{aligned}&\langle\Phi|\,a^+_{\nu_1}\,a^+_{\nu_2}\,a(x_2)\,a(x_1)\,H\,e^s|\Phi\rangle\\ &-\langle\Phi|H\,e^s|\Phi\rangle\,\langle\Phi|\,a^+_{\nu_1}\,a^+_{\nu_2}\,a(x_2)\,a(x_1)\,e^s|\Phi\rangle = 0,\end{aligned} \tag{2.10}$$

$$\begin{matrix}\cdot & \cdot & \cdot\\ \cdot\\ \cdot\end{matrix}$$

Equation (2.8) is named "one-particle-", (2.9) is named "two-particle-equation".

Solving this system of coupled equations we get the ground state energy E and the correlation functions $\Psi_1\ldots\Psi_n$ defined by

$$\begin{aligned}&\langle\Phi|\,a^+_{\nu_1}\ldots a^+_{\nu_n}\,a(x_1)\ldots a(x_n)|\Psi\rangle\\ &=:\langle x_1\ldots x_n|\Psi_n|\nu_1\ldots\nu_n\rangle\end{aligned} \tag{2.11}$$

closely related to the functions

$$\langle x_1\ldots x_n|S_n|\nu_1\ldots\nu_n\rangle$$

(see Ref. [1, 2, 7] and Chapter IV). Treating this system of equations one obtains finally

$$\begin{aligned} E = {} & \sum_{\nu} \langle \nu | T | \nu \rangle \\ & + \tfrac{1}{2} \sum_{\nu\nu'} \langle \nu\, \nu' | V \Psi_2 | \nu\, \nu' \rangle_{\mathrm{A}} \qquad (2.12) \end{aligned}$$

describing the groundstate energy

$$\begin{aligned} & \langle x_1 | T | \nu_1 \rangle - E \langle x_1 | \nu_1 \rangle \\ & + \sum_{\nu} \langle x_1\, \nu | T_2 \Psi_2 | \nu_1\, \nu \rangle_{\mathrm{A}} \qquad (2.12) \\ & + \sum_{\nu} \langle x_1\, \nu | V_{12} \Psi_2 | \nu_1\, \nu_2 \rangle_{\mathrm{A}} \\ & + \tfrac{1}{2} \sum_{\nu\nu'} \langle x_1\, \nu\, \nu' | V_{23} \Psi_3 | \nu_1\, \nu\, \nu' \rangle_{\mathrm{A}} = 0 \quad (2.13) \end{aligned}$$

for the "one-particle" equations and

$$\begin{aligned} & \langle x_1\, x_2 | (T_{x_1} + T_{x_2}) \Psi_2 | \nu_1\, \nu_2 \rangle_{\mathrm{A}} - E \langle x_1\, x_2 | \Psi_2 | \nu_1\, \nu_2 \rangle_{\mathrm{A}} \\ & + \langle x_1\, x_2 | V_{12} \Psi_2 | \nu_1\, \nu_2 \rangle_{\mathrm{A}} \\ & + \sum_{\nu} \langle x_1\, x_2\, \nu | T_3 \Psi_3 | \nu_1\, \nu_2\, \nu \rangle_{\mathrm{A}} \qquad (2.14) \\ & + \sum_{\nu} \langle x_1\, x_2\, \nu | V_{13} \Psi_3 | \nu_1\, \nu_2\, \nu \rangle_{\mathrm{A}} \\ & + \sum_{\nu} \langle x_1\, x_2\, \nu | V_{23} \Psi_3 | \nu_1\, \nu_2\, \nu \rangle_{\mathrm{A}} \\ & + \tfrac{1}{2} \sum_{\nu\nu'} \langle x_1\, x_2\, \nu\, \nu' | V_{34} \Psi_4 | \nu_1\, \nu_2\, \nu\, \nu' \rangle_{\mathrm{A}} = 0 \end{aligned}$$

for the "two-particle" equations.

We see that the one particle equations as well as the two particle equations are connected to "higher" correlations particularly by Ψ_3 and Ψ_4.

We want to restrict ourselves to one- and two-particle effects. Thus we have to neglect these higher correlations. Let us examine for example the matrix element $\langle x_1\, \nu\, \nu' | V_{23} \Psi_3 | \nu_1\, \nu\, \nu' \rangle_{\mathrm{A}}$ which couples the one-particle equation to the three-particle equations.

We decompose:

$$\begin{aligned} & \langle x_1\, \nu\, \nu' | V_{23} \Psi_3 | \nu_1\, \nu\, \nu' \rangle_{\mathrm{A}} \\ & \quad = \mathscr{A}_{\nu} [\langle x_1\, x_3 | V \Psi_2 | \nu_2\, \nu_3 \rangle \langle x_1 | \nu_1 \rangle] \\ & \qquad + \langle x_1\, x_2\, x_3 | V_{23}\, \chi_3^{(23)} | \nu_1\, \nu_2\, \nu_3 \rangle_{\mathrm{A}}\,, \\ & \langle x_1\, x_2\, x_3 | \chi_3^{(23)} | \nu_1\, \nu_2\, \nu_3 \rangle_{\mathrm{A}} \\ & \quad := \mathscr{A}_{\nu} [\langle x_1\, x_2 | S_2 | \nu_1\, \nu_2 \rangle \langle x_3 | \nu_3 \rangle] \\ & \qquad + \mathscr{A}_{\nu} [\langle x_1\, x_3 | S_2 | \nu_1\, \nu_3 \rangle \langle x_2 | \nu_2 \rangle] \\ & \qquad + \langle x_1\, x_2\, x_3 | S_3 | \nu_1\, \nu_2\, \nu_3 \rangle_{\mathrm{A}} \end{aligned}$$

($\mathscr{A}_{\nu}$ antisymmetrizes with respect to the ν).

Now we see that neglecting third order terms in this case means neglecting χ_3. This term is finite because the $V_{23} \Psi_3$ — as well as the $V\Psi_2$-term must be finite, while neglecting the S_3-term alone would lead to the infinite VS_2-terms.

III. Identical Particles

One can characterize a particle completely by specifying the kind of particle ($a, b, c, \ldots$) and the quantum mechanical state ($\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \ldots$) the particle occupies.

Definition. Two particles $a_{\boldsymbol{k}_1}$ and $b_{\boldsymbol{k}_2}$ are said to be identical, if there exists no observable operator which distinguishes the two particle state $|a_{\boldsymbol{k}_1}\, b_{\boldsymbol{k}_2}\rangle$ from the two-particle state $|b_{\boldsymbol{k}_1}\, a_{\boldsymbol{k}_2}\rangle$.

Another way to say this is the following: Two particles $a_{\boldsymbol{k}_1}$ and $b_{\boldsymbol{k}_2}$ are identical particles if the particles are able to exchange the particle states they are described by completely. This exchanging can be caused only by an interaction. So a third way to state this definition is: Two particles $a_{\boldsymbol{k}_1}$ and $b_{\boldsymbol{k}_2}$ are identical particles if there exists an interaction which puts particle a in that quantum mechanical state, in which particle b was before and vice versa. Being able to exchange their quantum mechanical states two particles lose their individuality. Any fictive marks used to distinguish the particles are exchanged too and become meaningless with respect to identifying the particles. Such particles have to be viewed as identical particles in (perhaps) different quantum mechanical states. A two-particle system consisting of two such fermions must therefore be antisymmetrized.

The interaction of resonances and nucleons is described by potentials having the form shown in Figure 1.

Consequently we have to antisymmetrize the N-N^* system. Equivalent to this is that the corre-

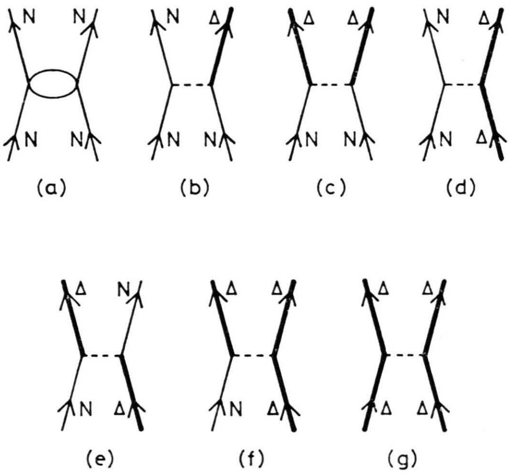

Fig. 1. The parts of the two body potential, needed to solve Eq. (7.15) (see Chapter VIII).

sponding creation and destruction operators must anticommute.

$$a_{N^*} a_N = -a_N a_{N^*}; \quad a^+_{N^*} a^+_N = -a^+_N a^+_{N^*}; \quad a_{N^*} a^+_N = -a^+_N a_{N^*} .$$

IV. The One- and Two-particle Equation

We are now prepared to derive a one- and two-particle equation describing the resonances explicitly. We want to restrict ourselves to two particle effects. So we start with an Hamiltonian containing only two particle forces.

$$H = \sum_{i=1}^{A} \{T_i + H_{\text{in}}(i)\} + \tfrac{1}{2} \sum_{i,j} V_{ij} . \tag{4.1}$$

T_i is the kinetic energy operator of particle i, H_{in} the so called intrinsic hamiltonian of particle i and V_{ij} the interaction operator of particle i and j. The eigenvalues of H_{in} are defined to be the resonance excitation energies.

$$H_{\text{in}} | i \rangle = 0 \quad \text{if } | i \rangle \text{ is a nucleonic state,} \tag{4.2}$$

$$H_{\text{in}} | N^* \rangle = (M^* - M) | N^* \rangle \quad \text{if } | N^* \rangle \text{ is a resonance state,} \tag{4.3}$$

M^* is the resonance mass, M the nucleon mass. Of course the following equation holds

$$\langle N_1^* | H_{\text{in}} | N_2^* \rangle = (M_2^* - M_1^*) \, \delta_{12} . \tag{4.4}$$

We demand a similar equation to be true with respect to the kinetic energy operator

$$\langle N_1^* | T | N_2^* \rangle = \langle N_1^* | T | N_1^* \rangle \, \delta_{12} . \tag{4.5}$$

The two particle interaction V_{ij} is given by a potential which in contrast to the "usual" nuclear physics contains inelastic channels and so permits transitions between different resonance states.

The "exact" ground state wave function of a nondegenerate A nucleon system $|\Psi\rangle$ is obtained by

$$|\Psi\rangle = \exp(R) \, |\Phi\rangle \tag{4.6}$$

from the shell model determinant $|\Phi\rangle$, with

$$R = \sum_{n=1}^{A} R_n , \tag{4.7}$$

and

$$R_n = \sum_{\substack{\varrho_1 \dots \varrho_n \\ \nu_1 \dots \nu_n}} a^+_{\varrho_1} \dots a^+_{\varrho_n} a_{\nu_n} \dots a_{\nu_1} \frac{1}{(n!)^2} \cdot \langle \varrho_1 \dots \varrho_n | R_{n,0} | \nu_1 \dots \nu_n \rangle$$

$$+ \sum_{i=1}^{k} \sum_{\substack{\Delta_1{}^i \varrho_2 \dots \varrho_n \\ \nu_1 \dots \nu_n}} a^+_{\Delta_1{}^i} a^+_{\varrho_2} \dots a^+_{\varrho_n} a_{\nu_n} \dots a_{\nu_1} \frac{1}{(n!)^2} \cdot \langle \Delta_1{}^i \varrho_2 \dots \varrho_n | R_{n,1} | \nu_1 \dots \nu_n \rangle$$

$$+ \cdots \tag{4.8}$$

$$\vdots$$

$$+ \sum_{i,j,\dots=1}^{k} \sum_{\substack{\Delta_1{}^i \dots \Delta_n{}^l \\ \nu_1 \dots \nu_n}} a^+_{\Delta_1{}^i} \dots a^+_{\Delta_n{}^l} a_{\nu_n} \dots a_{\nu_1} \frac{1}{(n!)^2} \cdot \langle \Delta_1{}^i \dots \Delta_n{}^l | R_{n,n} | \nu_1 \dots \nu_n \rangle .$$

The amplitudes $R_{n,j}$ are n-particle-n-hole excitation amplitudes including j resonances.

The creation and destruction operators obey anticommutation rules whether or not they belong to the same kind of resonances. Of course this leads straight forwardly to totally antisymmetrized wavefunctions.

If we characterize by $\delta_1 \dots \delta_n$ empty one-particle shell model states one may use the known formalism of the $\exp(S)$-theory.

$$|\Psi\rangle = \exp(S) \, |\Phi\rangle ; \quad S = \sum_{n=1}^{A} S_n ; \tag{4.9}$$

$$S_n = \sum_{\substack{\delta_1 \dots \delta_n \\ \nu_1 \dots \nu_n}} a^+_{\delta_1} \dots a^+_{\delta_n} a_{\nu_n} \dots a_{\nu_1} \frac{1}{(n!)^2} . \tag{4.10}$$

S_n are amplitudes now corresponding to n-particle and (not necessarily nucleonic) n-hole excitations.

We must be careful in doing this because the one particle states are qualitatively different in some respect from the old ones.

We define "reduced" wavefunctions by:

$$\langle 1 \dots n | \Psi_n | \nu_1 \dots \nu_n \rangle_{\text{A}} := \langle \Phi | a^+_{\nu_1} \dots a^+_{\nu_n} a(n) \dots a(1) | \Psi \rangle .$$

The creation- and destruction operators are connected by:

$$a^+(x) = \sum_{\alpha} \langle \alpha | x \rangle \, a_\alpha{}^+ ; \quad a(x) = \sum_{\alpha} \langle x | \alpha \rangle \, a_\alpha ;$$

$$a_\alpha = \int \mathrm{d}x \, \langle \alpha | x \rangle \, a(x) ;$$

$$a_\alpha{}^+ = \int \mathrm{d}x \, \langle x | \alpha \rangle \, a^+(x) ,$$

$|\alpha\rangle$ being some resonance- or nucleonic state. The Ψ_n can be expressed by the S_i just as is done in the "old" formalism.

$$\langle 1 \dots n | \Psi_n | \nu_1 \dots \nu_n \rangle_{\text{A}} = \sum_{\substack{\text{all terms of the} \\ \text{following structure}}} S_{\nu x} \mathscr{A}_\nu [\langle | S_i | \rangle \langle | S_j | \rangle \cdots \langle | S_m | \rangle] \quad i + j + \cdots + m = n \tag{4.11}$$

$$
\begin{aligned}
&+ S_{\nu x}\,\mathscr{A}_\nu[\langle 1 \dots s \,|\, \nu_1 \dots \nu^s\rangle \langle|S_j|\rangle \cdots \langle|S_m|\rangle] \\
&\qquad\qquad\qquad\qquad s + j + \dots + m = n \\
&+ \langle 1 \dots n \,|\, \nu_1 \dots \nu_n\rangle_{\mathrm{A}} \\
&+ \langle 1 \dots n \,|\, S_n \,|\, \nu_1 \dots \nu_n\rangle_{\mathrm{A}} \,. \qquad (4.11)
\end{aligned}
$$

For simplicity we define:

$$T_i + H_{\mathrm{in}}(i) =: \tilde{T}_i \,. \qquad (4.12)$$

The Hamiltonian $H = \tilde{T} + V$ can be written with the help of creation and destruction operators:

$$
\begin{aligned}
H = &\sum_{\alpha\beta} \langle \alpha \,|\, \tilde{T} \,|\, \beta\rangle\, a_\alpha^+ a_\beta^+ \\
&+ \tfrac{1}{2} \sum_{\substack{\alpha\beta \\ \gamma\delta}} \langle \alpha\,\beta \,|\, V \,|\, \gamma\,\delta\rangle\, a_\alpha^+ a_\beta^+ a_\delta a_\gamma; \qquad (4.13)
\end{aligned}
$$

α, β, γ, δ characterizes any one particle state. Demanding $\langle \Phi | \Psi\rangle = 1$ and $H|\Psi\rangle = E|\Psi\rangle$ it is a simple matter to show that:

$$
\begin{aligned}
E = &\sum_\nu \langle \nu \,|\, \tilde{T}\Psi_1 \,|\, \nu\rangle \\
&+ \tfrac{1}{2} \sum_{\nu\nu'} \langle \nu\,\nu' \,|\, V\Psi_2 \,|\, \nu\,\nu'\rangle_{\mathrm{A}} \,. \qquad (4.15)
\end{aligned}
$$

In Chapter II the following equation was called n-particle equation.

$$
\begin{aligned}
&\langle \Phi \,|\, a^+_{\nu_1} \dots a^+_{\nu_n}\, a(n) \dots a(1) \,|\, \Psi\rangle \qquad (4.16) \\
&\qquad = E\, \langle \Phi \,|\, a^+_{\nu_1} \dots a^+_{\nu_n}\, a(n) \dots a(1) \,|\, \Psi\rangle \,.
\end{aligned}
$$

We know $H = \tilde{T} + V$ and define: $\tilde{T}_x := \langle x \,|\, \tilde{T} \,|\, x\rangle$. As an example we manipulate the kinetic energy term.

$$\int \mathrm{d}x\, \tilde{T}_x \langle \Phi \,|\, a^+_{\nu_1} \dots a^+_{\nu_n}\, a(n) \dots a(1)\, a^+(x)\, a(x) \,|\, \Psi\rangle \,.$$

$a^+(x)\, a(x)$ is "shifted" now between $a^+_{\nu_n}$ and $a(n)$. Doing this leads to

$$
\begin{aligned}
&\int \mathrm{d}x\, \tilde{T}_x \langle \Phi \,|\, a^+_{\nu_1} \dots a^+_{\nu_n}\, a(n) \dots a(1)\, a^+(x)\, a(x) \,|\, \Psi\rangle \\
&\qquad = \sum_{i=1}^{n} \int \mathrm{d}x\, \tilde{T}_x \langle \Phi \,|\, a^+_{\nu_1} \dots a^+_{\nu_n}\, a(n) \dots a(1)\, \delta(x - x_i) \dots a(1) \,|\, \Psi\rangle \qquad (4.17) \\
&\qquad\quad + \int \mathrm{d}x\, \tilde{T}_x \langle \Phi \,|\, a^+_{\nu_1} \dots a^+_{\nu_n}\, a^+(x)\, a(x)\, a(n) \dots a(1) \,|\, \Psi\rangle \,.
\end{aligned}
$$

Using $a^+(x) = \sum_\alpha a_\alpha^+ \langle \alpha \,|\, x\rangle$ and defining

$$\langle 1 \dots n\,\nu \,|\, \tilde{T}_{n+1}\, \Psi_{n+1} \,|\, \nu_1 \dots \nu_n\,\nu\rangle := \int \mathrm{d}x\, \langle \nu \,|\, x\rangle\, \tilde{T}_x \langle 1 \dots n\,x \,|\, \Psi_{n+1} \,|\, \nu_1 \dots \nu_n\,\nu\rangle \qquad (4.18)$$

we finally get:

$$
\begin{aligned}
&\langle \Phi \,|\, a^+_{\nu_1} \dots a^+_{\nu_n}\, a(n) \dots a(1)\, \tilde{T} \,|\, \Psi\rangle \\
&\qquad = \sum_{i=1}^{n} \tilde{T}_i \langle 1 \dots n \,|\, \Psi_n \,|\, \nu_1 \dots \nu_n\rangle_{\mathrm{A}} + \sum_\nu \langle 1 \dots n\,\nu \,|\, \tilde{T}_{n+1}\, \Psi_{n+1} \,|\, \nu_1 \dots \nu_n\,\nu\rangle_{\mathrm{A}} \qquad (4.19)
\end{aligned}
$$

The potential energy term has to be treated the same way. Finally we get the n-particle equation in the following form

$$
\begin{aligned}
&\left(\sum_{i=1}^{n} \tilde{T}_i - E\right) \langle 1 \dots n \,|\, \Psi_n \,|\, \nu_1 \dots \nu_n\rangle_{\mathrm{A}} + \sum_{i<j\leqq n} V_{ij} \langle 1 \dots n \,|\, \Psi_n \,|\, \nu_1 \dots \nu_n\rangle_{\mathrm{A}} \\
&\quad + \sum_\nu \langle 1 \dots n\,\nu \,|\, \tilde{T}_{n+1}\, \Psi_{n+1} \,|\, \nu_1 \dots \nu_n\,\nu\rangle_{\mathrm{A}} + \sum_{i=1}^{n} \sum_\nu \langle 1 \dots n\,\nu \,|\, V_{i,\,n+1}\, \Psi_{n+1} \,|\, \nu_1 \dots \nu_n\,\nu\rangle_{\mathrm{A}} \qquad (4.20) \\
&\quad + \tfrac{1}{2} \sum_{\nu\nu'} \langle 1 \dots n\,\nu\,\nu' \,|\, V_{n+1,\,n+2}\, \Psi_{n+2} \,|\, \nu_1 \dots \nu_n\,\nu\,\nu'\rangle_{\mathrm{A}} = 0 \,.
\end{aligned}
$$

Restricting to $n = 1$ we obtain the one particle equation

$$
\begin{aligned}
&(\tilde{T}_1 - E) \langle 1 \,|\, \Psi_1 \,|\, \nu_1\rangle + \sum_\nu \langle 1\,\nu \,|\, \tilde{T}_2 \Psi_2 \,|\, \nu_1\,\nu\rangle_{\mathrm{A}} \\
&\qquad + \sum_\nu \langle 1\,\nu \,|\, V\Psi_2 \,|\, \nu_1\,\nu\rangle_{\mathrm{A}} + \tfrac{1}{2} \sum_{\nu\nu'} \langle 1\,\nu\,\nu' \,|\, V_{23}\Psi_3 \,|\, \nu_1\,\nu\,\nu'\rangle_{\mathrm{A}} = 0 \,. \qquad (4.21)
\end{aligned}
$$

We want to restrict ourselves to one and two particle effects. So we have to neglect the three particle effects connected by $V_{23}\Psi_3$ to the one particle equation.

Doing this and eliminating the macroscopic energy E by Eq. (4.15) we obtain the one particle equation:

$$\begin{aligned}\langle 1|\tilde{T}\Psi_1|\nu_1\rangle &- \sum_\nu \langle 1|\Psi_1|\nu\rangle\langle\nu|\tilde{T}\Psi_1|\nu_1\rangle \\ &+ \sum_\nu \langle 1\nu|\tilde{T}_2 S_2|\nu_1\nu\rangle_{\mathrm{A}} + \sum_\nu \langle 1\nu|V\Psi_2|\nu_1\nu\rangle_{\mathrm{A}} \\ &- \sum_{\nu\nu'} \langle 1|\Psi_1|\nu\rangle\langle\nu\nu'|V\Psi_2|\nu_1\nu'\rangle_{\mathrm{A}} = 0\,. \qquad (4.22)\end{aligned}$$

Next we engage in the two particle equation, i.e. $n=2$. Equation (4.20) reduces to.

$$\begin{aligned}(\tilde{T}_1 + \tilde{T}_2 - E)\langle 12|\Psi_2|\nu_1\nu_2\rangle_{\mathrm{A}} &+ \langle 12|V\Psi_2|\nu_1\nu_2\rangle_{\mathrm{A}} \\ &+ \sum_\nu \langle 12\nu|\tilde{T}_3\Psi_3|\nu_1\nu_2\nu\rangle_{\mathrm{A}} \\ &+ \sum_\nu \langle 12\nu|V_{13}\Psi_3|\nu_1\nu_2\nu\rangle_{\mathrm{A}} \qquad (4.23) \\ &+ \sum_\nu \langle 12\nu|V_{23}\Psi_3|\nu_1\nu_2\nu\rangle_{\mathrm{A}} \\ &+ \tfrac{1}{2}\sum_{\nu\nu'} \langle 12\nu\nu'|V_{34}\Psi_4|\nu_1\nu_2\nu\nu'\rangle_{\mathrm{A}} = 0\,.\end{aligned}$$

Again we have to neglect the three- and four particle effects given by Ψ_3 and Ψ_4.

Doing this and eliminating again the macroscopic energy E by Eq. (4.15) the two-particle equation (4.23) gets the following form

$$\begin{aligned}\langle 12|\overset{\Delta}{Q}(\tilde{T}_1 + \tilde{T}_2)\,S_2|\nu_1\nu_2\rangle_{\mathrm{A}} &+ \langle 12|\overset{\Delta}{Q}V\Psi_2|\nu_1\nu_2\rangle_{\mathrm{A}} \\ - \sum_\nu \{&\langle 12|S_2|\nu_1\nu_2\rangle_{\mathrm{A}}\langle\nu|\tilde{T}\Psi_1|\nu_1\rangle \\ &+ \langle 12|S_2|\nu_1\nu_2\rangle_{\mathrm{A}} * \langle\nu|\tilde{T}\Psi_1|\nu_2\rangle\} \\ - \sum_{\nu\nu'} \{&\langle 12|S_2|\nu\nu_2\rangle_{\mathrm{A}}\langle\nu\nu'|V\Psi_2|\nu_1\nu'\rangle_{\mathrm{A}} \qquad (4.24) \\ &+ \langle 12|S_2|\nu_1\nu'\rangle_{\mathrm{A}}\langle\nu\nu'|V\Psi_2|\nu\nu_2\rangle_{\mathrm{A}}\} \\ + \tfrac{1}{2}\sum_{\nu\nu'} &\langle 12|S_2|\nu\nu'\rangle_{\mathrm{A}}\langle\nu\nu'|V\Psi_2|\nu_1\nu_2\rangle_{\mathrm{A}} = 0\,.\end{aligned}$$

The operator $\overset{\Delta}{Q}$ is introduced for simplicity and is defined by: $\overset{\Delta}{Q} = (1 - \Psi_1(1))\cdot(1 - \Psi_1(2))$.

V. The One- and Two Particle Equations in Nuclear Matter

Nuclear matter is defined to be an infinit system. The uncorrelated ground state is build up by an equal number of protons and neutrons. The total momentum must be constant in time and we are allowed to put it zero. Consequently each slater determinant which occurs in any development of this ground state must have the same total momentum zero. Translational invariance of such a system causes the uncorrelated one particle wave functions to be momentum eigenstates. The "exact" A-particle state $|\Psi\rangle$ is described by the exp(S)-formalism by developing the uncorrelated ground state determinant $|\Phi\rangle$ with help of n-particle n-hole excitations.

$$|\Psi\rangle = \exp(S)\,|\Phi\rangle\,, \quad S = \sum_{n=1}^{A} S_n\,. \qquad (5.1)$$

We know that the A-particle state $|\tilde{\Phi}\rangle$ which is built up by $|\tilde{\Phi}\rangle = \exp(S_1)\,|\Phi\rangle$ as a part of (5.1) can be described again by a determinant. Consequently we always can find a ground state determinant $|\Phi\rangle$ in (5.1) which makes one-particle one-hole excitations in (5.1) redundant. In finite nuclei the determination of such a $|\Phi\rangle$ is äquivalent to the determination of the "best" shell model wave function. There it is a hard problem. Not so in nuclear matter.

We know the one particle wave functions to be plane waves and so are allowed from the beginning to put the nucleonic part of S_1 equal to zero. The attribute "nucleonic" in this context means that the particle influenced by S_1 remains a nucleon.

Let us first consider the one particle equation.

$$\begin{aligned}\langle 1|\tilde{T}\Psi_1|\nu_1\rangle &- \sum_\nu \langle 1|\Psi_1|\nu\rangle\langle\nu|\tilde{T}\Psi_1|\nu_1\rangle \\ &+ \sum_\nu \langle 1\nu|\tilde{T}_2 S_2|\nu_1\nu\rangle_{\mathrm{A}} + \sum_\nu \langle 1\nu|V\Psi_2|\nu_1\nu\rangle_{\mathrm{A}} \\ &- \sum_{\nu\nu'} \langle 1|\Psi_1|\nu\rangle\langle\nu\nu'|V\Psi_2|\nu_1\nu'\rangle_{\mathrm{A}} = 0\,. \qquad (5.2)\end{aligned}$$

The terms of (5.2) simplify in the following way.

$$\begin{aligned}\sum_\nu &\langle 1\nu|\tilde{T}_2 S_2|\nu_1\nu\rangle_{\mathrm{A}} \\ &= \sum_{\nu\alpha\beta} \langle 1\nu|\tilde{T}_2|\alpha\beta\rangle\langle\alpha\beta|S_2|\nu_1\nu\rangle_{\mathrm{A}} \qquad (5.3) \\ &= \sum_{\nu\alpha} \langle 1|\alpha\rangle\langle\nu|\tilde{T}|\nu\rangle\underbrace{\langle\alpha\nu|S_2|\nu_1\nu\rangle_{\mathrm{A}}}_{=0}\,,\end{aligned}$$

$$\begin{aligned}\sum_\nu &\langle 1\nu|V\Psi_2|\nu_1\nu\rangle_{\mathrm{A}} \\ &= \sum_{\nu\alpha\beta} \langle 1\nu|\alpha\beta\rangle\langle\alpha\beta|V\Psi_2|\nu_1\nu\rangle_{\mathrm{A}} \qquad (5.4) \\ &= \sum_{\nu\varkappa} \langle 1|\nu_1{}^{\varkappa}\rangle\langle\nu_1{}^{\varkappa}\nu|V\Psi_2|\nu_1\nu\rangle_{\mathrm{A}}\,.\end{aligned}$$

$\nu_1{}^{\varkappa}$ characterizes a one particle state with momentum ν_1. $\varkappa$ characterizes nucleonic — as well as resonance states. $\sum_{\varkappa}$ means summation over all intrinsic excitation states of the nucleon.

$$\begin{aligned}\sum_{\nu\nu'} &\langle 1|\Psi_1|\nu\rangle\langle\nu\nu'|V\Psi_2|\nu_1\nu'\rangle_{\mathrm{A}} \\ &= \sum_\nu \langle 1|\Psi_1|\nu_1\rangle\langle\nu_1\nu|V\Psi_2|\nu_1\nu\rangle_{\mathrm{A}}\,. \qquad (5.5)\end{aligned}$$

Thus the one particle equation is of the following form.

$$\begin{aligned}&\langle 1|\,\tilde{T}\Psi_1|\nu_1\rangle - \langle 1|\Psi_1|\nu_1\rangle\langle\nu_1|\,\tilde{T}\Psi_1|\nu_1\rangle\\&\quad+\sum_{\nu\varkappa}\langle 1|\nu_1{}^{\varkappa}\rangle\langle\nu_1{}^{\varkappa}\,\nu|\,V\Psi_2|\nu_1\,\nu\rangle_{\mathrm{A}}\qquad(5.6)\\&\quad-\sum_{\nu}\langle 1|\Psi_1|\nu_1\rangle\langle\nu_1\,\nu|\,V\Psi_2|\nu_1\,\nu\rangle_{\mathrm{A}}=0\,.\end{aligned}$$

We define $\nu_1{}^{\Delta}$ to be a resonance with momentum ν_1. Then we can eliminate the nucleonic part from the last two terms and obtain:

$$\begin{aligned}&\langle 1|\,\tilde{T}\Psi_1|\nu_1\rangle - \langle 1|\Psi_1|\nu_1\rangle\langle\nu_1|\,T\,|\nu_1\rangle\\&\quad+\sum_{\nu\Delta}\langle 1|\nu_1{}^{\Delta}\rangle\langle\nu_1{}^{\Delta}\,\nu|\,V\Psi_2|\nu_1\,\nu\rangle_{\mathrm{A}}\qquad(5.7)\\&\quad-\sum_{\nu}\langle 1|S_1|\nu_1\rangle\langle\nu_1\,\nu|\,V\Psi_2|\nu_1\,\nu\rangle_{\mathrm{A}}=0\,.\end{aligned}$$

Neglecting the resonances for the moment, Eq. (5.7) reduces to

$$\langle 1|\,T\,|\nu_1\rangle - \langle 1|\nu_1\rangle\langle\nu_1|\,T\,|\nu_1\rangle = 0\,, \qquad (5.8)$$

which proves that the $|\nu\rangle$ are momentum eigenstates.

Using (5.8) and defining an energy denominator by

$$\overset{\Delta}{e}_1 = \tilde{T} - \langle\nu_1|\,T\,|\nu_1\rangle - \sum_{\nu}\langle\nu_1\,\nu|\,V\Psi_2|\nu_1\,\nu\rangle_{\mathrm{A}} \quad (5.9)$$

one finally gets the following one-particle equation:

$$\langle 1|\overset{\Delta}{e}_1 S_1|\nu_1\rangle = -\sum_{\nu\Delta}\langle 1|\nu_1{}^{\Delta}\rangle\langle\nu_1{}^{\Delta}\,\nu|\,V\Psi_2|\nu_1\,\nu\rangle_{\mathrm{A}}\,. \qquad (5.10)$$

The two-particle equation (4.24) is simplified in the case of nuclear matter as follows:

1. The operator $\overset{\Delta}{Q}$ in $\langle 12|\overset{\Delta}{Q}(\tilde{T}_1+\tilde{T}_2)S_2|\nu_1\nu_2\rangle_{\mathrm{A}}$ disappears because S_2 projects out of the Fermi sea and $\tilde{T}_1$ as well as $\tilde{T}_2$ is diagonal in the one particle representation.

2. As the one particle states are momentum eigenstates, momentum conservation requires:

$$\langle\nu\,\nu'|\,V\Psi_2|\nu\,\nu_i\rangle_{\mathrm{A}} = \langle\nu\,\nu'|\,V\Psi_2|\nu\,\nu'\rangle_{\mathrm{A}}\,\delta_{\nu_i\nu'}\,.$$

Taking this into account the two particle equation obtains the following form:

$$\begin{aligned}&\langle 12|(\tilde{T}_1+\tilde{T}_2)\,S_2|\nu_1\,\nu_2\rangle_{\mathrm{A}} + \langle 12|\overset{\Delta}{Q}V\Psi_2|\nu_1\,\nu_2\rangle_{\mathrm{A}}\\&\quad-\langle 12|S_2|\nu_1\,\nu_2\rangle_{\mathrm{A}}\langle\nu_1|\,T\,|\nu_1\rangle\\&\quad-\langle 12|S_2|\nu_1\,\nu_2\rangle_{\mathrm{A}}\langle\nu_2|\,T\,|\nu_2\rangle\qquad(5.11)\\&\quad-\sum_{\nu}\{\langle 12|S_2|\nu_1\,\nu_2\rangle_{\mathrm{A}}\langle\nu_1\,\nu|\,V\Psi_2|\nu_1\,\nu\rangle_{\mathrm{A}}\\&\qquad+\langle 12|S_2|\nu_1\,\nu_2\rangle_{\mathrm{A}}\langle\nu\,\nu_2|\,V\Psi_2|\nu\,\nu_2\rangle_{\mathrm{A}}\}\\&\quad+\tfrac{1}{2}\sum_{\nu\nu'}\langle 12|S_2|\nu\,\nu'\rangle_{\mathrm{A}}\langle\nu\,\nu'|\,V\Psi_2|\nu_1\,\nu_2\rangle_{\mathrm{A}}=0\,.\end{aligned}$$

We define a two-particle energy denominator by

$$\begin{aligned}\overset{\Delta}{e}_2 = \tilde{T}_1+\tilde{T}_2 &- \langle\nu_1|\,T\,|\nu_1\rangle - \langle\nu_2|\,T\,|\nu_2\rangle\\&-\sum_{\nu}\langle\nu_1\,\nu|\,V\Psi_2|\nu_1\,\nu\rangle_{\mathrm{A}}\\&-\sum_{\nu}\langle\nu\,\nu_2|\,V\Psi_2|\nu\,\nu_2\rangle_{\mathrm{A}}\end{aligned}$$

and after adding $\langle 12|\overset{\Delta}{e}_2\Psi_1(1)\Psi_1(2)|\nu_1\nu_2\rangle_{\mathrm{A}}$ to Eq. (5.11) Eq. (5.11) assumes the following form:

$$\begin{aligned}&\langle 12|\overset{\Delta}{e}_2 S_2|\nu_1\,\nu_2\rangle_{\mathrm{A}} = \langle 12|\overset{\Delta}{e}_2\Psi_1(1)\,\Psi_1(2)|\nu_1\,\nu_2\rangle_{\mathrm{A}}\\&\quad-\langle 12|\overset{\Delta}{Q}V\Psi_2|\nu_1\,\nu_2\rangle_{\mathrm{A}}\qquad(5.12)\\&\quad-\tfrac{1}{2}\sum_{\nu\nu'}\langle 12|S_2|\nu\,\nu'\rangle_{\mathrm{A}}\langle\nu\,\nu'|\,V\Psi_2|\nu_1\,\nu_2\rangle_{\mathrm{A}}=0\,.\end{aligned}$$

We see that the one- and two-particle equation couple, because Ψ_1 occurs in Equation (5.12).

Let us have a look at this mechanism of coupling. According to the definitions we have made we can write:

$$\begin{aligned}&\langle 12|\overset{\Delta}{e}_2\Psi_1(1)\,\Psi_1(2)|\nu_1\,\nu_2\rangle_{\mathrm{A}}\\&\quad=\mathscr{A}_\nu[\langle 1|\,\tilde{T}_1\Psi_1|\nu_1\rangle\langle 2|\Psi_1|\nu_2\rangle]\\&\qquad+\mathscr{A}_\nu[\langle 1|\Psi_1|\nu_1\rangle\langle 2|\,\tilde{T}_1\Psi_1|\nu_2\rangle]\\&\qquad-\{\langle\nu_1|\,\tilde{T}\Psi_1|\nu_1\rangle+\langle\nu_2|\,\tilde{T}\Psi_1|\nu_2\rangle\qquad(5.13)\\&\qquad\quad+\sum_{\nu}\langle\nu_1\,\nu|\,V\Psi_2|\nu_1\,\nu\rangle_{\mathrm{A}}\\&\qquad\quad+\sum_{\nu}\langle\nu\,\nu_2|\,V\Psi_2|\nu\,\nu_2\rangle_{\mathrm{A}}\}\\&\qquad+\mathscr{A}_\nu[\langle 1|\Psi_1|\nu_1\rangle\langle 2|\Psi_1|\nu_2\rangle]\,.\end{aligned}$$

Doing the antisymmetrization with respect to the ν explicitly, some terms can be comprised by the use of the one particle equation.

Defining "one hole potentials" respectively "one hole transition potentials" by

$$U_{\nu_i} := \sum_{\nu}\langle\nu_i\,\nu|\,V\Psi_2|\nu_i\,\nu\rangle_{\mathrm{A}}\,, \qquad (5.14)$$

$$U_{\nu_i}{}^{\Delta} := \sum_{\nu}\langle\nu_i^{\Delta}\,\nu|\,V\Psi_2|\nu_i\,\nu\rangle_{\mathrm{A}}\,, \qquad (5.15)$$

we can write Eq. (5.13) in the following form:

$$\begin{aligned}&\langle 12|\overset{\Delta}{e}_2\Psi_1(1)\,\Psi_1(2)|\nu_1\,\nu_2\rangle_{\mathrm{A}}\qquad(5.16)\\&=-\langle 12|\nu_1\,\nu_2\rangle_{\mathrm{A}}\cdot(U_{\nu_1}+U_{\nu_2})\\&-\sum_{\Delta}\{\langle 12|\nu_1^{\Delta}\,\nu_2\rangle_{\mathrm{A}}\,U_{\nu_1^{\Delta}}+\langle 12|\nu_1\,\nu_2^{\Delta}\rangle_{\mathrm{A}}\,U_{\nu_2^{\Delta}}\}\\&-\{\langle 1|\nu_1\rangle\langle 2|S_1|\nu_2\rangle-\langle 2|\nu_1\rangle\langle 1|S_1|\nu_2\rangle\}\,U_{\nu_1}\\&+\{\langle 1|\nu_2\rangle\langle 2|S_2|\nu_1\rangle-\langle 2|\nu_2\rangle\langle 1|S_1|\nu_1\rangle\}\,U_{\nu_2}\\&-\sum_{\Delta}\{\langle 1|\nu_1^{\Delta}\rangle\langle 2|S_1|\nu_2\rangle-\langle 2|\nu_1^{\Delta}\rangle\langle 1|S_1|\nu_2\rangle\}\,U_{\nu_1^{\Delta}}\\&+\sum_{\Delta}\{\langle 1|\nu_2^{\Delta}\rangle\langle 2|S_1|\nu_1\rangle-\langle 2|\nu_2^{\Delta}\rangle\langle 1|S_1|\nu_1\rangle\}\,U_{\nu_2^{\Delta}}\,.\end{aligned}$$

Using the one-particle equation, the S_1-terms of Eq. (5.16) can be eliminated in favour of one hole-transition potential terms.

The one particle equation

$$\langle 1 | \overset{\Delta}{e}_1 S_1 | \nu_1 \rangle = - \sum_{\nu \Delta} \langle 1 | \nu_1^\Delta \rangle \langle \nu_1^\Delta \nu | V \Psi_2 | \nu_1 \nu \rangle_{\mathrm{A}} \tag{5.17}$$

can be written as

$$\langle 1 | \alpha \rangle \langle \alpha | \overset{\Delta}{e}_1 | \beta \rangle \langle \beta | S_1 | \nu_1 \rangle = - \sum_{\Delta} \langle 1 | \nu_1^\Delta \rangle U_{\nu_1^\Delta} . \tag{5.18}$$

S_1 concerns only resonance excitation. Momentum conservation has to be required. So we get:

$$\sum_{\Delta} \langle 1 | \nu_1^\Delta \rangle \{ \langle \nu_1^\Delta | \tilde{T} - T_{\nu_1} - U_{\nu_1} | \nu_1^\Delta \rangle \cdot \langle \nu_1^\Delta | S_1 | \nu_1 \rangle + U_{\nu_1^\Delta} \} = 0 . \tag{5.19}$$

$\sum_{\Delta}$ sums up linearly independent terms. So Eq. (5.19) can finally be written:

$$\langle \nu_1^\Delta | S_1 | \nu_1 \rangle = - U_{\nu_1^\Delta} / (T_{\nu_1^\Delta} - T_{\nu_1} - U_{\nu_1}) . \tag{5.20}$$

It is a simple matter to eliminate the S_1-terms of (5.16) with the help of Equation (5.20). We obtain the two particle equation in the following form:

$$\begin{aligned}
\langle 12 | \overset{\Delta}{e}_2 \psi_2 | \nu_1 \nu_2 \rangle_{\mathrm{A}} = & - \langle 12 | \nu_1 \nu_2 \rangle_{\mathrm{A}} \cdot (U_{\nu_1} + U_{\nu_2}) \\
& - \sum_{\Delta} \langle 12 | \nu_1^\Delta \nu_2 \rangle_{\mathrm{A}} U_{\nu_1^\Delta} \left(1 - \frac{U_{\nu_2}}{T_{\nu_1^\Delta} - T_{\nu_1} - U_{\nu_1}} \right) \\
& - \sum_{\Delta} \langle 12 | \nu_1 \nu_2^\Delta \rangle_{\mathrm{A}} U_{\nu_2^\Delta} \left(1 - \frac{U_{\nu_1}}{T_{\nu_2^\Delta} - T_{\nu_2} - U_{\nu_2}} \right) \\
& + \sum_{\Delta_1 \Delta_2} \langle 12 | \nu_1^\Delta \nu_2^\Delta \rangle_{\mathrm{A}} U_{\nu_1^\Delta} U_{\nu_2^\Delta} \left(\frac{1}{T_{\nu_1^\Delta} - T_{\nu_1} - U_{\nu_1}} + \frac{1}{T_{\nu_2^\Delta} - T_{\nu_2} - U_{\nu_2}} \right) \\
& - \langle 12 | \overset{\Delta}{Q} V \Psi_2 | \nu_1 \nu_2 \rangle_{\mathrm{A}} - \tfrac{1}{2} \sum_{\nu \nu'} \langle 12 | S_2 | \nu \nu' \rangle \langle \nu \nu' | V \Psi_2 | \nu_1 \nu_2 \rangle_{\mathrm{A}} .
\end{aligned} \tag{5.21}$$

VI. Deduction of a Computable Integro-differential Equation-system from the Generalized Bethe-Goldstone Equation

The fact that we are confronted with terms containing one-hole-transition-potentials, the $U_{\nu_i^\Delta}$ makes the treatment of the two particle equation (5.21) very hard.

Fortunately we are able to proove that such potentials are only possible if we are concerned with resonances having the same quantum numbers as the nucleon. As we are restricting ourselves to the most important resonance, the Δ(1236)-resonance, we prove that an $U_{\nu_i^{\Delta(1236)}}$ does not occur. We have defined

$$U_{\nu_i^\Delta} := \sum_{\nu} \langle \nu_i^\Delta \nu | V \Psi_2 | \nu_i \nu \rangle_{\mathrm{A}} . \tag{6.1}$$

$|\nu\rangle$ is a momentum-eigenstate and therefore characterized by the momentum $\boldsymbol{p}$, the isotopic spin $(t\, t_3)$ and the spin $(s\, m_s)$

$$| \nu \rangle \triangleq | \boldsymbol{p} ; t\, t_3 ; s\, m_s \rangle .$$

Concerning $|\nu_i^\Delta\rangle$ the same is true

$$| \nu_i^\Delta \rangle \triangleq | \boldsymbol{p}_i ; t_{\Delta_i} t_{\Delta_i}^3 ; s_i\, m_{s_i} \rangle .$$

We can prove $U_{\Delta_i^{\Delta(1236)}}$ to be zero by restricting ourselves to the isotopic-part of the matrix-element $\langle \nu_i^\Delta \nu | V \Psi_2 | \nu_i \nu \rangle_{\mathrm{A}}$. So we have to prove:

$$\sum_{t_3} \langle \tfrac{3}{2} t_{\Delta_i}^3 ; \tfrac{1}{2} t_3 | V \Psi_2 | \tfrac{1}{2} t_3{}^i ; \tfrac{1}{2} t_3 \rangle \overset{!}{=} 0 . \tag{6.2}$$

V as well as Ψ_2 are diagonal with respect to the total isotopic spin. For this it is useful to write:

$$\sum_{t_3} \langle \tfrac{3}{2} t_{\Delta_i}^3 ; \tfrac{1}{2} t_3 | V \Psi_2 | \tfrac{1}{2} t_3{}^i ; \tfrac{1}{2} t_3 \rangle = \sum_{T M t_3} \langle \tfrac{3}{2} t_{\Delta_i}^3 ; \tfrac{1}{2} t_3 | T M \rangle \cdot \langle T M | V \Psi_2 | T M \rangle \langle T M | \tfrac{1}{2} t_3{}^i ; \tfrac{1}{2} t_3 \rangle . \tag{6.3}$$

From this expression it is straightforward to show that:

$$\begin{aligned}
& \sum_{t_3} \langle \tfrac{3}{2} t_{\Delta_i}^3 ; \tfrac{1}{2} t_3 | V \Psi_2 | \tfrac{1}{2} t_3{}^i ; \tfrac{1}{2} t_3 \rangle \\
& = \sum_{T} \frac{1}{\sqrt{2T+1}} \langle T \| V \Psi_2 \| T \rangle (-1) \cdot \frac{2T+1}{2 \cdot \frac{3}{2} + 1} \delta_{3/2\, 1/2}\, \delta t_{\Delta_i}^3 t_3{}^i = 0 .
\end{aligned} \tag{6.4}$$

The two-particle-equation now gets the following form:

$$\begin{aligned}
\langle 12 | \overset{\Delta}{e}_2 \Psi_2 | \nu_1 \nu_2 \rangle_{\mathrm{A}} = & - \langle 12 | \nu_1 \nu_2 \rangle_{\mathrm{A}} \cdot (U_{\nu_1} + U_{\nu_2}) \\
& - \langle 12 | \overset{\Delta}{Q} V \Psi_2 | \nu_1 \nu_2 \rangle_{\mathrm{A}} \\
& - \tfrac{1}{2} \sum_{\nu \nu'} \langle 12 | S_2 | \nu \nu' \rangle_{\mathrm{A}} \langle \nu \nu' | V \Psi_2 | \nu_1 \nu_2 \rangle_{\mathrm{A}} .
\end{aligned} \tag{6.5}$$

We want to prepare this equation for numerical calculation. To do this it is suitable to introduce a new quantum number n which characterizes the one particle components the two particle state is made of.

We define:

$n = 1 =$ N-N-state,
$n = 2 =$ N-Δ(1236)-state,
$n = 3 = \Delta$(1236)-Δ(1236)-state.

$\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ characterize one-particle space coordinates. Then we write the two-particle-equation:

$$\begin{aligned}&\langle n; \boldsymbol{r}_1 \boldsymbol{r}_2 | \overset{\Delta}{e}_2 \Psi_2 | 1; \nu_1 \nu_2 \rangle_{\mathrm{A}}\\ &\quad = -\langle n; \boldsymbol{r}_1 \boldsymbol{r}_2 | 1; \nu_1 \nu_2 \rangle_{\mathrm{A}} \cdot (U_{\nu_1} + U_{\nu_2})\\ &\qquad - \langle n; \boldsymbol{r}_1 \boldsymbol{r}_2 | \overset{\Delta}{Q} V \Psi_2 | 1; \nu_1 \nu_2 \rangle_{\mathrm{A}}\\ &\qquad - \langle n; \boldsymbol{r}_1 \boldsymbol{r}_2 | S_2 \Pi V \Psi_2 | 1; \nu_1 \nu_2 \rangle_{\mathrm{A}}\,.\end{aligned} \tag{6.6}$$

The operator Π is defined by

$$\Pi := \tfrac{1}{2} \sum_{\nu\nu'} | 1; \nu\, \nu' \rangle_{\mathrm{A}} \langle 1\, \nu\, \nu' |\,;$$

$\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ characterize one-particle momentums and m_1 and m_2 one-particle restmasses.

We define total and relative space and momentum operators by:

$$\begin{aligned}\boldsymbol{R} &= (m_1 \boldsymbol{r}_1 + m_2 \boldsymbol{r}_2)/(m_1 + m_2)\,;\\ \boldsymbol{r} &= \boldsymbol{r}_1 - \boldsymbol{r}_2\,,\\ \boldsymbol{K} &= \boldsymbol{k}_1 + \boldsymbol{k}_2\,;\\ \boldsymbol{k} &= (m_1 \boldsymbol{k}_2 - m_2 \boldsymbol{k}_1)/(m_1 + m_2)\,.\end{aligned} \tag{6.7}$$

In nuclear matter the total momentum of the correlated wave-function must be the same as for the plane wave. This means that the dependence of the correlated wave function on $\boldsymbol{R}$ and $\boldsymbol{K}$ is just the same as that of the uncorrelated plane wave. Neglecting for the moment spin and isotopic spin the two particle equation can be written in the following form.

$$\begin{aligned}&\langle n; \boldsymbol{r} | \overset{\Delta}{e}_2 \Psi_2 | 1; \boldsymbol{k}_0 \rangle \langle \boldsymbol{R} | \boldsymbol{K}_0 \rangle\\ &\quad = -\langle n; \boldsymbol{r} | 1; \boldsymbol{k}_0 \rangle \langle \boldsymbol{R} | \boldsymbol{K}_0 \rangle (U_{\nu_1} + U_{\nu_2})\\ &\qquad - \langle n; \boldsymbol{r} | \overset{\Delta}{Q} V \Psi_2 | 1; \boldsymbol{k}_0 \rangle \langle \boldsymbol{R} | \boldsymbol{K}_0 \rangle\\ &\qquad - \langle n; \boldsymbol{r} | S_2 \Pi V \Psi_2 | 1; \boldsymbol{k}_0 \rangle \langle \boldsymbol{R} | \boldsymbol{K}_0 \rangle\,.\end{aligned} \tag{6.8}$$

The two particle energy denominator $\overset{\Delta}{e}_2$ was defined:

$$\overset{\Delta}{e}_2 = \tilde{T}_1 + \tilde{T}_2 - T_{\nu_1} - T_{\nu_2} - U_{\nu_1} - U_{\nu_2}\,.$$

Paying attention to total momentum conservation this can be written explicitly — after decomposition into radial and angular momentum parts

$$\begin{aligned}\overset{\Delta}{e}_2 = \Delta M &- \frac{1}{2m} \frac{1}{r} \frac{\mathrm{d}^2}{\mathrm{d}r^2} r + \frac{1}{2m} \frac{l(l+1)}{r^2}\\ &- \frac{k_0^2}{2m} - U_{\nu_1} - U_{\nu_2}\,.\end{aligned}$$

Consequently the terms of (6.8) due to the total momentum separate totally. Before we continue to treat the two particle equation it is necessary to write the two-particle states in different way. We do this by the following replacement:

$$\langle n; \boldsymbol{r} | \rightarrow \langle n (t_1 t_2) t; r (l' s') j m_j |\,.$$

The particular meaning is the following:

t_i = isotopic spin of particle i,
t = total isotopic spin of the two-particle-state,
r = relativ distance,
l' = relative angular momentum,
s' = total spin of the two-particle system,
j = total angular momentum,
m_j = projection of total angular momentum,

and

$$| 1; \boldsymbol{k}_0 \rangle \rightarrow | 1 (\tfrac{1}{2} \tfrac{1}{2}) t; h_0 \hat{h}_0 s m_s \rangle\,.$$

The particular meaning is the following:

h_0 = relative momentum,
$\hat{h}_0$ = angle of relative momentum,
s = total spin of the two particle system,
m_s = projection of the total spin.

Now the two-particle equation has the following form:

$$\begin{aligned}&\langle n; (t_1 t_2) t; r (l' s') j m_j | \overset{\Delta}{e}_2 \Psi_2 \underset{\uparrow}{|} 1 (\tfrac{1}{2} \tfrac{1}{2}) t; h_0 \hat{h}_0 s m_s \rangle\\ &\qquad - \langle n; (t_1 t_2) t; r (l' s') j m_j | 1 (\tfrac{1}{2} \tfrac{1}{2}) t; h_0 \hat{h}_0 s m_s \rangle (U_{\nu_1} + U_{\nu_2})\\ &\qquad - \langle n; (t_1 t_2) t; r (l' s') j m_j | \overset{\Delta}{Q} V \Psi_2 \underset{\uparrow}{|} 1 (\tfrac{1}{2} \tfrac{1}{2}) t; h_0 \hat{h}_0 s m_s \rangle\\ &\qquad - \langle n; (t_1 t_2) t; r (l' s') j m_j | S_2 \Pi V \Psi_2 \underset{\uparrow}{|} 1 (\tfrac{1}{2} \tfrac{1}{2}) t; h_0 \hat{h}_0 s m_s \rangle\,.\end{aligned} \tag{6.9}$$

By inserting the "unit" operator

$$\sum_l \int \mathrm{d}r'\, r'^2 \,|\, 1\,(\tfrac{1}{2}\,\tfrac{1}{2})\,t; r'\,(l\,s)\,j\,m_j\rangle \langle 1\,(\tfrac{1}{2}\,\tfrac{1}{2})\,t; r'\,(l\,s)\,j\,m_j\,|$$

in (6.9) at the positions indicated by ↑, using

$$\begin{aligned}&\langle n\,(t_1\,t_2)\,t; r\,(l'\,s)\,j\,m_j\,|\, 1\,(\tfrac{1}{2}\,\tfrac{1}{2})\,t; h_0\,\hat{h}_0\,s\,m_s\rangle \\ &\qquad = \delta_{n,1}\,\delta_{s,s'} \sum_m \langle j\,m_j\,|\,l'\,s\,m\,m_s\rangle\, 4\pi\, i^{l'}\, j_{l'}(h_0\,r)\; Y_{l'}^{m*}(\hat{h}_0)\,,\end{aligned}$$

multiplying with $Y_{l_1}^{m_1}(\hat{h}_0)$ and integrating over $\hat{h}_0$, one gets:

$$\begin{aligned}&\langle n; (t_1\,t_2)\,t; r\,(l'\,s')\,j\,m_j\,|\,\overset{\Delta}{e}_2 \Psi_2\,|\,1\,(\tfrac{1}{2}\,\tfrac{1}{2})\,t; r\,(l\,s)\,j\,m_j\rangle \cdot j_l(h_0\,r) \\ &\quad = -\, j_l(h_0\,r)\,(U_{\nu_1} + U_{\nu_2})\,\delta_{n,1}\,\delta_{ll'} \\ &\qquad - \int \mathrm{d}r'\, r'^2 \langle n; (t_1\,t_2)\,t; r\,(l'\,s')\,j\,m_j\,|\,\underset{\uparrow}{\overset{\Delta}{Q}V}\Psi_2\,|\,1\,(\tfrac{1}{2}\,\tfrac{1}{2})\,t; r'\,(l\,s)\,j\,m_j\rangle\, j_l(h_0\,r) \\ &\qquad - \int \mathrm{d}r'\, r'^2 \langle n; (t_1\,t_2)\,t; r\,(l'\,s')\,j\,m_j\,|\,S_2 \underset{\uparrow}{}\Pi \underset{\uparrow}{} V \underset{\uparrow}{}\Psi_2\,|\,1\,(\tfrac{1}{2}\,\tfrac{1}{2})\,t; r'\,(l\,s)\,j\,m_j\rangle\, j_l(h_0\,r)\,.\end{aligned} \tag{6.10}$$

The next steps to do are the following ones: First we know that S_1 is equal to zero. Consequently symbolically $\Psi_2 = 1 + S_2$. For convenience we define $\overset{\Delta}{P} := 1 - \overset{\Delta}{Q}$. Then we insert the above mentioned "unit" operator at the positions indicated by ↑ and multiply with r. Next we define relative-wave functions by:

$${}^{mjt}_{\;n}\mathsf{U}^{(t_3 t_4)(l_2 S_2)}_{(t_1 t_2)(l_1 S_1)}(h_0\,, r) := \{r \cdot \langle m\,(t_3\,t_4)\,t; r\,(l_2\,S_2)\,j\,m_j\,|\,\Psi_2\,|\,n\,(t_1\,t_2)\,t; r\,(l_1\,S_1)\,j\,m_j\rangle \cdot j_{l_1}(h_0\,r)\}$$

and characterize the V-matrix elements by

$${}^{mjt}_{\;n}\mathsf{V}^{(t_3 t_4)(l_2 S_2)}_{(t_1 t_2)(l_1 S_1)}(r) := \langle m\,(t_3\,t_4)\,t; r\,(l_2\,S_2)\,j\,m_j\,|\,V\,|\,n\,(t_1\,t_2)\,t; r\,(l_1\,S_1)\,j\,m_j\rangle\,.$$

Last we insert the following momentum "unit" operators:

$$\frac{1}{(2\pi)^3} \sum_{m_s} \int k^2\, \mathrm{d}k\, \mathrm{d}\hat{h}\,|\,n\,(t_1\,t_2)\,t; h\,\hat{h}\,s\,m_s\rangle \langle n\,(t_1\,t_2)\,t; h\,\hat{h}\,s\,m_s\,|\,.$$

This is necessary because the Pauli projection operators are diagonal only in momentum space. Proceeding in this way the two particle equation assumes the following form:

$$\begin{aligned}&r \cdot \overset{\Delta}{e}_2 \cdot \frac{1}{r} \cdot {}^{njt}_{\;1}\mathsf{U}^{(t_1 t_2)(l' s')}_{(\frac{1}{2}\frac{1}{2})(ls)}(h_0\,, r) = -\, r \cdot j_l(h_0\,r) \cdot (U_{\nu_1} + U_{\nu_2})\,\delta_{n,1}\,\delta_{l,l'} - \sum_{\substack{l_1 S_1\\ m t_3 t_4}} {}^{njt}_{\;m}\mathsf{V}^{(t_1 t_2)(l' s')}_{(t_3 t_4)(l_1 S_1)}(r) \\ &\quad \cdot {}^{mjt}_{\;1}\mathsf{U}^{(t_3 t_4)(l_1 S_1)}_{(\frac{1}{2}\frac{1}{2})(ls)}(h_0 . r) + \sum_{\substack{l_1 S_1 m_s'\\ m t_3 t_4}} \frac{1}{(2\pi)^3} \int \mathrm{d}r' \int h^2\, \mathrm{d}h\, \mathrm{d}\hat{h}\; r \cdot r' \\ &\quad * \langle n\,(t_1\,t_2)\,t; r\,(l'\,s')\,j\,m_j\,|\,n\,(t_1\,t_2)\,t; h\,\hat{h}\,s'\,m_s{}'\rangle \cdot \overset{\Delta}{P}{}^n(h, K) \\ &\quad * \langle n\,(t_1\,t_2)\,t; h\,\hat{h}\,s'\,m_s{}'\,|\,n\,(t_1\,t_2)\,t; r'\,(l'\,s')\,j\,m_j\rangle * {}^{njt}_{\;m}\mathsf{V}^{(t_1 t_2)(l' s')}_{(t_3 t_4)(l_1 S_1)}(r) \cdot {}^{mjt}_{\;1}\mathsf{U}^{(t_3 t_4)(l_1 S_1)}_{(\frac{1}{2}\frac{1}{2})(ls)}(h_0\,, r) \\ &\quad - \sum_{\substack{l_1 S_1\\ m t_3 t_4}} \sum_{\substack{l_2 S_2 m S_1\\ w t_5 t_6}} \frac{1}{(2\pi)^3} \int \mathrm{d}r' \int h^2\, \mathrm{d}h\, \mathrm{d}\hat{h}\; r \cdot r' * \langle n\,(t_1\,t_2)\,t; r\,(l'\,s')\,j\,m_j\,|\,\Psi_2\,|\,m\,(t_3\,t_4)\,t; r\,(l_1\,S_1)\,j\,m_j\rangle \\ &\quad * \langle m\,(t_3\,t_4)\,t; r\,(l_1\,S_1)\,j\,m_j\,|\,m\,(t_3\,t_4)\,t; h\,\hat{h}\,S_1\,m_{S_1}\rangle \cdot \Pi(h, K) * \delta_{m,1}\,\delta_{t_3\frac{1}{2}}\,\delta_{t_4\frac{1}{2}} \\ &\quad \cdot \langle m\,(t_3\,t_4)\,t; h\,\hat{h}\,s_1\,m_{S_1}\,|\,m\,(t_3\,t_4)\,t; r'\,(l_1\,S_1)\,j\,m_j\rangle * {}^{mjt}_{\;w}\mathsf{V}^{(t_3 t_4)(l_1 S_1)}_{(t_5 t_6)(l_2 S_2)}(r) \cdot {}^{wjt}_{\;1}\mathsf{U}^{(t_5 t_6)(l_2 S_2)}_{(\frac{1}{2}\frac{1}{2})(ls)}(h_0\,, r') \\ &\quad + \sum_{\substack{l_1 S_1 m_s'\\ m t_3 t_4}} \frac{1}{(2\pi)^3} \int \mathrm{d}r' \int h^2\, \mathrm{d}h\, \mathrm{d}\hat{h}\; r \cdot r' * \langle n\,(t_1\,t_2)\,t; r\,(l'\,s')\,j\,m_j\,|\,n\,(t_1\,t_2)\,t; h\,\hat{h}\,s'\,m_s{}'\rangle \cdot \Pi(h, K)\end{aligned} \tag{6.11}$$

$$* \delta_{n,1}\, \delta_{t_2 \frac{1}{2}} \cdot \langle n(t_1 t_2)\, t; h\, \hat{h}\, s'\, m_s' \,|\, n(t_1 t_2)\, t; r'(l'\, s')\, j\, m_j \rangle * {}^{njt}_{\;m}\mathsf{V}^{(t_1t_2)(l's')}_{(t_3t_4)(l_1S_1)}(r')$$
$$\cdot\, {}^{mjt}_{\;\;1}\mathsf{U}^{(t_3t_4)(l_1S_1)}_{(\frac{1}{2}\frac{1}{2})(ls)}(h_0, r')\,.$$

$\overset{\varDelta}{P}{}^n(h, K)$ is defined by

$$\overset{\varDelta}{P}{}^n(h, K) := \langle n(t_1 t_2)\, t; h\, \hat{h}\, s\, m_s \,|\, \overset{\varDelta}{P} \,|\, n(t_1 t_2)\, t; h\, \hat{h}\, s\, m_s \rangle\,.$$

$\Pi(h, K)$ is defined by

$$\Pi(h, K) := \langle n(t_1 t_2)\, t; h\, \hat{h}\, s\, m_s \,|\, \Pi \,|\, n(t_1 t_2)\, t; h\, \hat{h}\, s\, m_s \rangle\,.$$

The fact that $\overset{\varDelta}{P}{}^n(h, K)$ as well as $\Pi(h, K)$ depend on the total momentum K is justified by the angle averaging method discussed in Chapter VIIc. Using

$$\langle n(t_1 t_2)\, t; r(l\, s)\, j\, m_j \,|\, n(t_1 t_2)\, t; h\, \hat{h}\, s\, m_s \rangle = \langle j\, m_j \,|\, l\, s\, m_j - m_s; m_s \rangle \cdot 4\pi \cdot i^l j_l(h\, r) \cdot Y_l^{m_j - m_s}(\hat{h})$$

the "plane wave" terms in (6.11) can be expanded. Integrating $\hat{h}$ and summing up the spin projections we get the following two particle equation:

$$\begin{aligned}
r \cdot \overset{\varDelta}{e}_2 \frac{1}{r}\, {}^{njt}_{\;\;1}\mathsf{U}^{(t_1t_2)(l's')}_{(\frac{1}{2}\frac{1}{2})(ls)}(h_0, r') = &- r\, j_l(h_0 \cdot r) \cdot (U_{\nu_1} + U_{\nu_2})\, \delta_{n,1}\, \delta_{ll'} - \sum_{\substack{l_1 S_1\\ m t_3 t_4}} {}^{njt}_{\;m}\mathsf{V}^{(t_1t_2)(l's')}_{(t_3t_4)(l_1S_1)}(r) \cdot {}^{mjt}_{\;\;1}\mathsf{U}^{(t_3t_4)(l_1S_1)}_{(\frac{1}{2}\frac{1}{2})(ls)}(h_0, r)\\
&+ \frac{1}{2\pi^2} \int h^2\, \mathrm{d}h\, r\, j_{l'}(h\, r) \cdot \overset{\varDelta}{P}{}^n(h, K) \cdot 4\pi \cdot \sum_{\substack{l_1 S_1\\ m t_3 t_4}} \int r'\, \mathrm{d}r' * j_{l'}(h\, r') \cdot {}^{njt}_{\;m}\mathsf{V}^{(t_1t_2)(l's')}_{(t_3t_4)(l_1S_1)}(r') \cdot {}^{mjt}_{\;\;1}\mathsf{U}^{(t_3t_4)(l_1S_1)}_{(\frac{1}{2}\frac{1}{2})(ls)}(h_0, r')\\
&- \frac{1}{2\pi^2} \sum_{l_1 S_1} \int h^2\, \mathrm{d}h \cdot {}^{njt}_{\;\;1}\mathsf{U}^{(t_1t_2)(l's')}_{(\frac{1}{2}\frac{1}{2})(l_1S_1)}(h, r) \cdot \Pi(h, K) * 4\pi \cdot \sum_{\substack{l_2 S_2\\ w t_5 t_6}} \int r'\, \mathrm{d}r'\, j_{l_1}(h\, r')\\
&\quad \cdot {}^{1jt}_{\;w}\mathsf{V}^{(\frac{1}{2}\frac{1}{2})(l_1S_1)}_{(t_5t_6)(l_2S_2)}(r') \cdot {}^{wjt}_{\;\;1}\mathsf{U}^{(t_5t_6)(l_2S_2)}_{(\frac{1}{2}\frac{1}{2})(ls)}(h_0, r')\\
&+ \frac{1}{2\pi^2} \int h^2\, \mathrm{d}h\, r \cdot j_{l'}(h\, r) \cdot \Pi(h, K) \cdot 4\pi \cdot \sum_{\substack{l_1 S_1\\ m t_3 t_4}} \int r'\, \mathrm{d}r'\, j_{l'}(h\, r') * {}^{njt}_{\;m}\mathsf{V}^{(t_1t_2)(l's')}_{(t_3t_4)(l_1S_1)}(r')\\
&\quad \cdot {}^{mjt}_{\;\;1}\mathsf{U}^{(t_3t_4)(l_1S_1)}_{(\frac{1}{2}\frac{1}{2})(ls)}(h_0, r')\, \delta_{n,1}\, \delta_{t_1 \frac{1}{2}}\, \delta_{t_2 \frac{1}{2}}\,.
\end{aligned} \tag{6.12}$$

Some definitions help to simplify this equation with respect to the numerical calculation. First we define a T-matrix by:

$${}^{mjt}_{\;\;1}\mathsf{T}^{(t_3t_4)(l_1S_1)}_{(\frac{1}{2}\frac{1}{2})(ls)}(h, h_0) := 4\pi \cdot \sum_{l_2 S_2 w t_5 t_6} \int r'\, \mathrm{d}r'\, j_{l_1}(h\, r') * {}^{mjt}_{\;w}\mathsf{V}^{(t_3t_4)(l_1S_1)}_{(t_5t_6)(l_2S_2)}(r') \cdot {}^{wjt}_{\;\;1}\mathsf{U}^{(t_5t_6)(l_2S_2)}_{(\frac{1}{2}\frac{1}{2})(ls)}(h_0, r') \tag{6.13}$$

and a F-matrix by:

$$\begin{aligned}
{}^{njt}_{\;\;1}\mathsf{F}^{(t_1t_2)(l's')}_{(\frac{1}{2}\frac{1}{2})(ls)}(h_0, K, r) := &- \frac{1}{2\pi^2} \int h^2\, \mathrm{d}h\, r \cdot j_{l'}(h\, r) \cdot \overset{\varDelta}{P}{}^n(h, K) * {}^{njt}_{\;\;1}\mathsf{T}^{(t_1t_2)(l's')}_{(\frac{1}{2}\frac{1}{2})(ls)}(h, h_0)\\
&+ \frac{1}{2\pi^2} \sum_{l_1 S_1} \int h^2\, \mathrm{d}h\, {}^{njt}_{\;\;1}\mathsf{U}^{(t_1t_2)(l's)}_{(\frac{1}{2}\frac{1}{2})(l_1S_1)}(h, r) * \Pi(h, K)\, {}^{1jt}_{\;\;1}\mathsf{T}^{(\frac{1}{2}\frac{1}{2})(l_1S_1)}_{(\frac{1}{2}\frac{1}{2})(ls)}(h, h_0)\\
&- \frac{1}{2\pi^2} \int h^2\, \mathrm{d}h\, r \cdot j_{l'}(h\, r) * \Pi(h, K)\, {}^{njt}_{\;\;1}\mathsf{T}^{(t_1t_2)(l's')}_{(\frac{1}{2}\frac{1}{2})(ls)}(h, h_0)\, \delta_{n,1}\, \delta_{t_1 \frac{1}{2}}\, \delta_{t_2 \frac{1}{2}}\,.
\end{aligned} \tag{6.14}$$

If we use units such that $\hbar^2/2m = 1$ and remember that

$$\overset{\varDelta}{e}_2 = -\frac{1}{2m} \frac{1}{r} \frac{d^2}{\mathrm{d}r^2} r + \frac{1}{2m} \frac{l(l+1)}{r^2} + \varDelta M - \frac{k_0{}^2}{2m} - U_{\nu_1} - U_{\nu_2}$$

we get the following two particle equation:

$$\begin{aligned}&\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2}-\frac{l'(l'+1)}{r^2}+h_0{}^2+U_{\nu_1}+U_{\nu_2}-\Delta M\right]\cdot {}^{njt}_{\;1}\mathsf{U}^{(t_1t_2)(l's')}_{(\frac{1}{2}\frac{1}{2})(ls)}(h_0,r)\\&\quad-\sum_{l_1S_1mt_3t_4}{}^{njt}_{\;m}\mathsf{V}^{(t_1t_2)(l's')}_{(t_3t_4)(l_1S_1)}(r)\cdot{}^{mjt}_{\;1}\mathsf{U}^{(t_3t_4)(l_1S_1)}_{(\frac{1}{2}\frac{1}{2})(ls)}(h_0,r)\\&\quad=r\cdot j_l(h_0r)\cdot(U_{\nu_1}+U_{\nu_2})\,\delta_{n,1}\,\delta_{ll'}+{}^{njt}_{\;1}\mathsf{F}^{(t_1t_2)(l's')}_{(\frac{1}{2}\frac{1}{2})(ls)}(h_0,K,r)\,.\end{aligned}\tag{6.15}$$

It is easy to see that (6.15) is a system of coupled inhomogenous integro differential equations.

VII. Equation for the One-hole Potential, the Defect Parameter χ and the Modified Pauli Operator for Numerical Calculation

a) The One-hole Potential

In Chapter V we have defined the one hole potential by:

$$U_{\nu_1}:=\sum_{\nu}\langle\nu_1\,\nu|\,V\Psi_2\,|\,\nu_1\,\nu\rangle_{\mathrm{A}}\,.\tag{7.1}$$

Using the ideas and conventions of Chapter VI it is a simple matter to see that $\langle\nu_1\,\nu|\,V\Psi_2\,|\,\nu_1\,\nu\rangle$ can be written in the following form:

$$\begin{aligned}&\langle\nu_1\,\nu|\,V\Psi_2\,|\,\nu_1\,\nu\rangle\\&\quad=\sum_{lj}(2l+1)\,|\langle l\,s\,0\,m_s\,|\,j\,m_s\rangle|^2\\&\quad\cdot{}^{1jt}_{\;1}\mathsf{T}^{(\frac{1}{2}\frac{1}{2})(ls)}_{(\frac{1}{2}\frac{1}{2})(ls)}(h_0,h_0)\,.\end{aligned}\tag{7.2}$$

The next step we have to do is to follow an idea of Brueckner and Gammel [8].

Particle ν_1 as well as particle ν may be protons (+) or neutrons (—), their spins may be "up" (α) or "down" (β). Suppose particle ν to be a proton it's spin being up. Coupling the one-particle spins and isotopic spins to total spins and isotopic spins (7.1) assumes the following form:

$$\begin{aligned}U_{\nu_1}=&\\\sum_{\nu}\{&\langle\nu_1\,\nu|\,V\Psi_2(s=1,m_s=1,T=1,T_z=1)\,|\,\nu_1\,\nu\rangle\\&+\tfrac{1}{2}\langle\nu_1\,\nu|\,V\Psi_2(s=1,m_s=0,T=1,T_z=1)\,|\,\nu_1\,\nu\rangle\\&+\tfrac{1}{2}\langle\nu_1\,\nu|\,V\Psi_2(s=0,m_s=0,T=1,T_z=1)\,|\,\nu_1\,\nu\rangle\\&+\tfrac{1}{2}\langle\nu_1\,\nu|\,V\Psi_2(s=1,m_s=1,T=1,T_z=0)\,|\,\nu_1\,\nu\rangle\\&+\tfrac{1}{2}\langle\nu_1\,\nu|\,V\Psi_2(s=1,m_s=1,T=0,T_z=0)\,|\,\nu_1\,\nu\rangle\\&+\tfrac{1}{4}\langle\nu_1\,\nu|\,V\Psi_2(s=1,m_s=0,T=1,T_z=0)\,|\,\nu_1\,\nu\rangle\\&+\tfrac{1}{4}\langle\nu_1\,\nu|\,V\Psi_2(s=0,m_s=0,T=1,T_z=0)\,|\,\nu_1\,\nu\rangle\\&+\tfrac{1}{4}\langle\nu_1\,\nu|\,V\Psi_2(s=1,m_s=0,T=0,T_z=0)\,|\,\nu_1\,\nu\rangle\\&+\tfrac{1}{4}\langle\nu_1\,\nu|\,V\Psi_2(s=0,m_s=0,T=0,T_z=0)\,|\,\nu_1\,\nu\rangle\}-\\&\qquad\text{exchange terms.}\end{aligned}\tag{7.3}$$

If we now take into account that

1. the one-hole-potential depends on m_s only by the Clebsch-Gordan-coefficients,
2. the exchange-terms just contribute a factor two, according to the Pauli-principle.
3. $V\Psi_2$ is independent from T_z and
4. ν_1 must not be a proton with spin "up"

we get:

$$\begin{aligned}U(k_1)&=\sum_{kTsjlm_s}\frac{2T+1}{4}\langle k_1\,k|\,V\Psi_2(s\,m_s\,T)\,|\,k_1\,k\rangle_{\mathrm{A}}\\&=\sum_{kTsjl}\frac{(2T+1)(2j+1)}{2}\,{}^{1jt}_{\;1}\mathsf{T}^{(\frac{1}{2}\frac{1}{2})(ls)}_{(\frac{1}{2}\frac{1}{2})(ls)}(h_0,h_0)\,.\end{aligned}\tag{7.4}$$

Only antisymmetric terms are summed up in (7.4). The one hole potential is presented to be a function of $|\boldsymbol{k}_1|$. All other quantum numbers are summed over. The reason for this is, that the one-hole potential is only used to compute the one-hole-energies of the energy-denominator, and the one hole-energies depend on $|\boldsymbol{k}_1|$ only. The seeming discrepancy that $U(k_1)$ is a function depending on $|\boldsymbol{k}_1|$, whereas the T-matrix depends on the relative momentum h_0, is resolved as follows: The T-matrix is computed with respect to an uncorrelated two particles tate $|\nu_1\nu_2\rangle$. Because of the Pauli projection operators the T-matrix depends on the relativ momentum $\boldsymbol{h}_0$ as well as on the total momentum $\boldsymbol{K}_0$. Therefore we identify:

$$\begin{gathered}{}^{1jt}_{\;1}\mathsf{T}^{(\frac{1}{2}\frac{1}{2})(ls)}_{(\frac{1}{2}\frac{1}{2})(ls)}(h_0,h_0)\equiv{}^{1jt}_{\;1}\mathsf{T}^{(\frac{1}{2}\frac{1}{2})(ls)}_{(\frac{1}{2}\frac{1}{2})(ls)}(k_1,k_2,\cos\vartheta)\,;\\\vartheta=\sphericalangle(\boldsymbol{k}_1,\boldsymbol{k}_2)\,.\end{gathered}$$

The one-hole-potential becomes

$$\begin{aligned}U(k_1)&=\sum_{kTsjl}\frac{(2T+1)(2j+1)}{2}\\&\quad{}^{1jt}_{\;1}\mathsf{T}^{(\frac{1}{2}\frac{1}{2})(ls)}_{(\frac{1}{2}\frac{1}{2})(ls)}(k_1,k,\cos\vartheta)\\&=\sum_{Tsjl}\frac{(2T+1)(2j+1)}{2}\cdot\frac{1}{(2\pi)^3}\int_0^{k_F}h^2\,\mathrm{d}h\end{aligned}\tag{7.5}$$

$$
\begin{aligned}
&* \int_0^{\pi} \sin\vartheta \, \mathrm{d}\vartheta \cdot {}^{1jt}_{\;1}\mathsf{T}^{(\frac{1}{2}\frac{1}{2})(ls)}_{(\frac{1}{2}\frac{1}{2})(ls)}(k_1, k, \cos\vartheta) \int_0^{2\pi} \mathrm{d}\varphi \\
&= \frac{1}{\pi^2} \sum_{Tsjl} \frac{(2T+1)(2j+1)}{8} \int_0^{k_\mathrm{F}} h^2 \, \mathrm{d}h \\
&* \int_0^{\pi} \sin\vartheta \, \mathrm{d}\vartheta \cdot {}^{1jt}_{\;1}\mathsf{T}^{(\frac{1}{2}\frac{1}{2})(ls)}_{(\frac{1}{2}\frac{1}{2})(ls)}(k_1, k, \cos\vartheta) \, .
\end{aligned}
$$

If the one-hole-potential is known, it is a simple matter to compute the potential- and binding energy per particle. We present the result without proof.

$$
E_\mathrm{Bin}/N = \frac{3}{10} k_\mathrm{F}^2 + \frac{3}{2 k_\mathrm{F}^3} \cdot \int_0^{k_\mathrm{F}} U(k) \, k^2 \, \mathrm{d}k \, .
$$

b) The Defect-Parameter $\varkappa$

The defect-parameter $\varkappa$ is defined by:

$$
\varkappa := \frac{1}{N} \sum_{\nu_1 \nu_2} \langle \nu_1 \nu_2 | (\Psi_2 - 1)^+ (\Psi_2 - 1) | \nu_1 \nu_2 \rangle_\mathrm{A} \, . \tag{7.6}
$$

This parameter is of interest because of the following reason.

If one writes down the potential energy with the help of diagrammatic methods as a sum over all connected diagrams and then arrange these diagrams with respect to the number of independent hole lines one is able to prove the following fact. A diagram B differing from a diagram A only by containing one more independent hole line yields a contribution to the potential energy which is reduced to that of diagram A by a factor $\varkappa$ [12]. Of course it is very important if one sums up such diagram classes that $\varkappa$ is considerably smaller than one.

The same formalism and the same arguments as in the case of the one-hole-potential lead us from (7.6) to the following computable form:

$$
\begin{aligned}
\varkappa = {} & \frac{3}{\pi^2 \cdot k_\mathrm{F}^3} \int_0^{k_\mathrm{F}} k_1^2 \, \mathrm{d}k_1 \int_0^{k_\mathrm{F}} k_2^2 \, \mathrm{d}k_2 \int_0^{\pi} \sin\vartheta \, \mathrm{d}\vartheta \\
& * \sum_{Tsjl} \frac{(2T+1)(2j+1)}{8} \sum_{l's'wt_1t_2} 4\pi \qquad (7.7) \\
& * \int \mathrm{d}r \, \{ {}^{wjt}_{\;\;1}\mathsf{U}^{(t_1t_2)(l's')}_{(\frac{1}{2}\frac{1}{2})(ls)}(h_0, r) \\
& - r \, j_l(h_0 r) \, \delta_{1w} \, \delta_{ll'} \, \delta_{ss'} \}^2 \, .
\end{aligned}
$$

Just as mentioned above in the case of the one-hole-potential the difficulty in Eq. (7.7) which occurs by integrating one-particle coordinates while the integrand depends on relative coordinates is solved. Using the same arguments as in the case of the one-hole-potentials the integrand can be interpreted as a function depending on one particle coordinates.

c) The Modified Pauli Projection Operators

The Bethe-Goldstone equation contains a projection operator which ensures that the ground state determinant $|\Phi\rangle$ does not occur as an intermediate state. The two particle equation (6.15) generalized by the $\exp(S)$-formalism contains in addition a projection operator Π which ensures that the operator S_2 operates on occupied states.

$$
\overset{\Delta}{Q}{}^n = \sum_{\varrho_1 \varrho_2} | \varrho_1 \varrho_2 \rangle \langle \varrho_1 \varrho_2 | \, .
$$

ϱ_1 and ϱ_2 characterize empty one particle shell model states. ϱ_1 or ϱ_2 may be nucleonic — as well as resonance states.

$$
\Pi = \sum_{\nu_1 \nu_2} | \nu_1 \nu_2 \rangle \langle \nu_1 \nu_2 | \, .
$$

ν_1 and ν_2 characterize occupied (nucleonic) one particle shell model states.

The numerical calculation is done to a great part in relative and center of mass coordinates. Thus the exact treatment of the projection operators would require an enormous amount of numerically work, because $\overset{\Delta}{Q}{}^n$ as well as Π depend in a non trivial way as well on $|\boldsymbol{K}|$ as on $|\boldsymbol{h}|$ and $\sphericalangle(\boldsymbol{h}, \boldsymbol{K})$. To compute these operators we use the angle averaging approximation, described before by many authors [9, 10], so that we content ourselves to present the results.

I. The operator $\overset{\Delta}{Q}{}^n$ $(n = 1)$; nucleon-nucleon-system.

1) $\overset{\Delta}{Q}{}^1 = 0$ if $k \leqq \sqrt{k_\mathrm{F}^2 - \frac{1}{4} K^2}$,

2) $\overset{\Delta}{Q}{}^1 = 1$ if $k > k_\mathrm{F} + \frac{1}{2} K$,

3) $\overset{\Delta}{Q}{}^1 = (h^2 + \frac{1}{4} K^2 - k_\mathrm{F}^2)/K \cdot h$ else.

II. The operator $\overset{\Delta}{Q}{}^n$ $(n = 2)$; resonance-nucleon-system.

1) $\overset{\Delta}{Q}{}^2 = 0$ if $k_\mathrm{F} \geqq \frac{1}{2} K + h$,

2) $\overset{\Delta}{Q}{}^2 = 1$ if $h > k_\mathrm{F} + \frac{1}{2} K$,

3) $\overset{\Delta}{Q}{}^2 = \frac{1}{2} + (h^2 + \frac{1}{4} K^2 - k_\mathrm{F}^2)/K \cdot h$ else.

III. The operator $\overset{\Delta}{Q}{}^n$ $(n = 3)$ resonance-resonance-system.

$\overset{\Delta}{Q}{}^3 = 1$ in all cases.

IV. The operator Π:

1) $\Pi = 1$ if $k_F \geqq \frac{1}{2}K + h$,

2) $\Pi = 0$ if $h > \sqrt{k_F^2 - \frac{1}{4}K^2}$,

3) $\Pi = (k_F^2 - \frac{1}{4}K^2 - h^2)/K \cdot h$ else.

VIII. Two Particle Potentials

The explicit form and a discussion of our potentials is given by Niephaus in [11].

To compute the two-particle equation (6.15) we need two-particle interaction potentials which include the $\varDelta$(1236)-resonance explicitly. The graphs shown in Fig. 1 may characterize the necessary parts of the potentials. The total interaction potential we have to think of as a sum of all these contributions.

It is possible to describe the pure nucleon-nucleon-potential (a) by a phenomenological Ansatz. The free parameters are fitted in order to reproduce the experimental two particle results. The Reid-soft-core-potential is an example. All other potentials (b)—(g) which involve at least one resonance in the entrance — and (or) final — channel can't be constructed in a similar way. The necessary experimental data are not available. These potentials must be constructed from a more theoretical point of view.

The potentials we use are one-boson-exchange potentials. That means the interaction is caused by the exchange of π, ϱ, ω, ... mesons.

Of course it is not possible to use the Reid-soft-core-potential in its well known form to compute the two particle equation (6.15) which takes resonance-nucleon as well as resonance-resonance configurations into account explicitly. The phenomenological Reid potential includes the resonance-effects. A calculation of Eq. (6.15) using the unmodified Reid-potential would cause double counting of resonance effects. We eliminate this problem of double counting the following way. The two-nucleon-scattering as well as the deuteron is calculated by taking the resonances into consideration explicitly. The free parameters of the Reid-softcore-potential are refitted in order to reproduce the experimental two particle dates. This means that the resonances taken into consideration explicitly are eliminated from the Reid-potential.

IX. Numerical Calculations

If we have a look at Eq. (6.15) we see that the energy denominator depends not only on the relative momentum $|\boldsymbol{h}_0|$ but via the one-hole-potential as well on the amount of the one particle momenta $|\boldsymbol{k}_i|$ of the nucleons interacting in the Fermi sea. Moreover, the F-matrix is not determined by $|\boldsymbol{h}_0|$ alone but depends — via the Pauli-projection operators — on $|\boldsymbol{K}_0|$ as well. This is remarkable because the center of mass motion can be separated completely from the two-particle equation. This center of mass dependence which can be observed in the energy-denominator as well as in the Pauli projection operators is taken into account correctly .

We describe this briefly because our procedure does not employ approximations used normally. Moreover it leads to a one-hole-potential directly.

Although Eq. (6.15) must be solved in the relativ and center of mass system — otherwise it is impossible to separate the center of mass motion — the numerical calculation is started from one particle coordinates. We start from two nucleons in the Fermi sea with definite one particle momentum $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$. These one particle momentums are transformed the relative and center of mass system and only when this is done Eq. (6.15) is solved. Thus the G-matrix-element we obtain as a result of our calculations is diagonal with respect to the one-particle momenta $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$. $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ are varied such that the whole Fermi sea is exhausted symmetrically. For symmetry reasons it suffices to vary the one-particle momenta on the surface of a semicircle. Figure 2 illustrates our procedure. $|\boldsymbol{k}_1|$ and $|\boldsymbol{k}_2|$ are varied such that the interval $[0, k_F]$ is devided into five equal parts and the angle $\vartheta \in [0, \pi]$ is devided into nine equal parts. As the G-matrix is symmetrical with respect to $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ the semicircle is exhausted by $\boldsymbol{k}_1$ as well as by $\boldsymbol{k}_2$. By rotation of the semicircle the whole

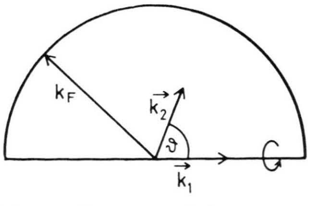

Fig. 2. Variation of the one particle momenta $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ in the Fermi sea (see Chapter IX).

Fermi sea is exhausted. This treatment of the intervals $[0, k_F]$ and $[0, \pi]$ turned out to be numerically stable.

After Eq. (6.15) is solved 105 times with respect to the different pairs $(\boldsymbol{k}_1, \boldsymbol{k}_2)$, a one hole potential is used in a next iteration to calculate an improved energy denominator. This sequence of iterations is controlled by the one-hole-potential itself and is stopped if the one-hole-potential changes from one to the next iteration by less than one percent. The F-matrix in (6.15) contains two terms which enclude the Π-projector explicitly and which are called $S_2{}^2$-terms. The exact calculation of these terms which generalize the Bethe-Goldstone-equation is difficult for the following reasons. Calculation of the $S_2{}^2$-terms requires the integration of the wave-function over the relative momentum $|\boldsymbol{h}|$. As the center of mass momentum $\boldsymbol{K}$ is given definitely by the two one-particle momenta $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ we start from and the interaction potentials don't change this total momentum, this integration has to be carried out with $\boldsymbol{K}$ being a constant. Just this is the difficulty. If we solve Eq. (6.15) selfconsistently as described above, neglecting the $S_2{}^2$-terms in the first step, we get wave functions corresponding to the 105 different pairs of one-particle momenta $(\boldsymbol{k}_1, \boldsymbol{k}_2)$. However, these wave functions are not suitable for calculating the $S_2{}^2$-terms because they correspond to different relative momenta $|\boldsymbol{h}|$ as well as to different center of mass momenta $|\boldsymbol{K}|$. Because we have been very interested in calculating Eq. (6.15) as accurate as possible we have solved this problem in the following way. The computed wave functions corresponding to one pair $(\boldsymbol{k}_1, \boldsymbol{k}_2)$ are represented by a polynomial of degree six. The six coefficients which determine one partial wave function in space coordinates can be represented by a product polynomial of degree 5×5 in relative and center of mass momentum space very nicely, after the wave functions have been calculated with respect to all pairs $(\boldsymbol{k}_1, \boldsymbol{k}_2)$.

Alltogether this procedure gives us an analytical form of each partial wave function. Knowing this analytical form it is a simple matter to calculate the $S_2{}^2$-terms correctly. Finally we have to do an iteration procedure to calculate the one-hole-potential selfconsistently by a calculation involving the $S_2{}^2$-terms. Computing Eq. (6.15) the only approximation we have done is the angle-averaging approximation concerning the Pauli-projection operators. To study the saturation properties we have calculated self consistently the binding energy as a function of seven different Fermi momenta out of $[0.96\,\text{fm}^{-1}, 1.56\,\text{fm}^{-1}]$. In doing so we studied the following four different cases:

C) Nuclear matter calculation without any resonances, neglecting the $S_2{}^2$-terms.

D) Nuclear matter calculation without any resonances; $S_2{}^2$-terms are involved.

A) Nuclear matter calculation with resonances involved; neglecting the $S_2{}^2$-terms.

B) Nuclear matter calculation with resonances involved; $S_2{}^2$-terms are involved.

We give all available values of these four cases in Tables 1 to 7.

The one-hole potential described in Chapter 7 can be represented by a polynomial of degree two.

$$U(k) = X + Y \cdot k^2 ,$$

k characterizes a one particle momentum $|\boldsymbol{k}|$. The coefficients X and Y very strongly depend on the Fermi momentum k_F. They can be represented by a polynominal of degree three. Together, we can write

$$U(k) = a_1 + a_2 \cdot k_F + a_3 k_F{}^2 + a_4 \cdot k_F{}^3 + (b_1 + b_2 \cdot k_F + b_3 \cdot k_F{}^2 + b_4 \cdot k_F{}^3) \cdot k^2 .$$

Table 8 contains the numerical values of the parameters — corresponding to fully selfconsistent one-hole-potentials.

X. Conclusions, Discussion of the Results

Before we go on discussing which conclusions can be drawn from our numerical results, especially concerning the two body potentials, we summarize the main features of our paper.

The exp(S)-formalism, which allows the ground state description of a nondegenerate many-body-system, is modified in such a way, that the explicit treatment of the Δ(1236)-states becomes possible. This formalism which develops the exact ground state $|\Psi\rangle$ with respect to n-particle-n-hole excitations of a shell-modell-determinant permits a description of resonances in formal analogy to excited one particle shell model states. By restriction to two particle interactions and by description of these interactions with nonrelativistic potentials

	A	B	C	D
Potential Energy	-16.90	-17.35	-18.26	-18.74
Kinetic Energy	11.46	11.46	11.46	11.46
Binding energy	-5.43	-5.89	-6.79	-7.28
Kappa	0.161	0.175	0.093	0.106
Resonance admixture	4.40	4.47		

Table 1. Results corresponding to the different sets of partial waves.

Fermi momentum: KF = 0.96 fm^{-1}
Density: RHO = 0.06 fm^{-3}
Average interparticle spacing: RM = 1.59 fm

Partial-wave	Kappa			
	A	B	C	D
$^1S_0$ N-N	0.013	0.013	0.012	0.012
$^5D_0$ N-Δ	0.031	0.029		
$^1S_0$ Δ-Δ	0.002	0.002		
$^5D_0$ Δ-Δ	0.005	0.005		
Binding energy	-6.85	-6.65	-7.68	-7.68

Partial-wave	Kappa			
	A	B	C	D
$^1P_1$ N-N	0.001	0.001	0.001	0.001
$^1P_1$ Δ-Δ	0.000	0.000		
$^5P_1$ Δ-Δ	0.000	0.000		
$^5F_1$ Δ-Δ	0.000	0.000		
Binding energy	0.42	0.42	0.46	0.46

Partial-wave	Kappa			
	A	B	C	D
$^3P_0$ N-N	0.001	0.001	0.001	0.001
$^3P_0$ N-Δ	0.001	0.001		
$^3P_0$ Δ-Δ	0.000	0.000		
$^7F_0$ Δ-Δ	0.000	0.000		
Binding energy	-1.16	-1.16	-1.19	-1.19

Partial-wave	Kappa			
	A	B	C	D
$^3P_1$ N-N	0.003	0.003	0.002	0.002
$^3P_1$ N-Δ	0.000	0.000		
$^5P_1$ N-Δ	0.003	0.003		
$^5F_1$ N-Δ	0.001	0.001		
$^7F_1$ Δ-Δ	0.000	0.000		
Binding energy	2.92	2.91	2.60	2.60

Partial-wave	Kappa			
	A	B	C	D
$^3D_2$ N-N	0.000	0.000	0.000	0.000
$^3D_2$ Δ-Δ	0.000	0.000		
$^7D_2$ Δ-Δ	0.000	0.000		
$^7G_2$ Δ-Δ	0.000	0.000		
Binding energy	-1.01	-1.01	-0.99	-0.99

Partial-wave	Kappa			
	A	B	C	D
$^3S_1$ N-N	0.020	0.021	0.019	0.019
$^3D_1$ N-N	0.054	0.069	0.056	0.069
$^3S_1$ Δ-Δ	0.002	0.002		
$^3D_1$ Δ-Δ	0.001	0.001		
$^7D_1$ Δ-Δ	0.012	0.013		
$^7G_1$ Δ-Δ	0.001	0.001		
Binding enery	-9.31	-10.01	-9.54	-10.07

Partial-wave	Kappa			
	A	B	C	D
$^3D_1$ N-N	0.000	0.000	0.000	0.000
$^3S_1$ N-N	0.000	0.001	0.000	0.001
$^3S_1$ Δ-Δ	0.000	0.000		
$^3D_1$ Δ-Δ	0.000	0.000		
$^7D_1$ Δ-Δ	0.000	0.000		
$^7G_1$ Δ-Δ	0.000	0.000		
Binding energy	0.34	0.37	0.34	0.37

Partial-wave	Kappa			
	A	B	C	D
$^1D_2$ N-N	0.000	0.000	0.000	0.000
$^5S_2$ N-Δ	0.000	0.000		
$^5D_2$ N-Δ	0.000	0.000		
$^5G_2$ N-Δ	0.000	0.000		
$^1D_2$ Δ-Δ	0.000	0.000		
$^5S_2$ Δ-Δ	0.000	0.000		
$^5D_2$ Δ-Δ	0.000	0.000		
$^5G_2$ Δ-Δ	0.000	0.000		
Binding energy	-0.53	-0.53	-0.55	-0.55

Partial-wave	Kappa			
	A	B	C	D
$^3P_2$ N-N	0.001	0.001	0.001	0.001
$^3F_2$ N-N	0.001	0.001	0.001	0.001
$^3P_2$ N-Δ	0.001	0.001		
$^3F_2$ N-Δ	0.000	0.000		
$^5P_2$ N-Δ	0.002	0.002		
$^5F_2$ N-Δ	0.003	0.003		
Binding-energy	-1.63	-1.61	-1.61	-1.60

Partial-wave	Kappa			
	A	B	C	D
$^3F_2$ N-N	0.000	0.000	0.000	0.001
$^3P_2$ N-N	0.000	0.000	0.000	0.000
$^3P_2$ N-Δ	0.000	0.000		
$^3F_2$ N-Δ	0.000	0.000		
$^5P_2$ N-Δ	0.000	0.000		
$^5F_2$ N-Δ	0.000	0.000		
Binding energy	-0.09	-0.09	-0.09	-0.09

	A	B	C	D
Potential Energy	-20.09	-20.69	-22.25	-22.84
Kinetic Energy	13.98	13.98	13.98	13.98
Binding energy	-6.11	-6.71	-8.27	-8.87
Kappa	0.188	0.204	0.099	0.11
Resonance admixture	5.57	5.59		

Table 2. Results corresponding to the different sets of partial waves.

Fermi momentum: KF = 1.06 fm^{-1}
Density: RHO = 0.08 fm^{-3}
Average interparticle spacing: RM = 1.44 fm

Partial-wave	Kappa			
	A	B	C	D
$^1S_0$ N-N	0.016	0.016	0.014	0.013
$^5D_0$ N-Δ	0.039	0.036		
$^1S_0$ Δ-Δ	0.002	0.002		
$^5D_0$ Δ-Δ	0.006	0.006		
Binding energy	-8.28	-8.03	-9.52	-9.52

Partial-wave	Kappa			
	A	B	C	D
$^1P_1$ N-N	0.002	0.002	0.002	0.002
$^1P_1$ Δ-Δ	0.000	0.000		
$^5P_1$ Δ-Δ	0.000	0.000		
$^5F_1$ Δ-Δ	0.000	0.000		
Binding energy	0.67	0.68	0.72	0.72

Partial-wave	Kappa			
	A	B	C	D
$^3P_0$ N-N	0.001	0.001	0.001	0.001
$^3P_0$ N-Δ	0.001	0.001		
$^3P_0$ Δ-Δ	0.000	0.000		
$^7F_0$ Δ-Δ	0.000	0.000		
Binding energy	-1.58	-1.58	-1.64	-1.64

Partial-wave	Kappa			
	A	B	C	D
$^3P_1$ N-N	0.005	0.005	0.002	0.002
$^3P_1$ N-Δ	0.000	0.000		
$^5P_1$ N-Δ	0.004	0.004		
$^5F_1$ N-Δ	0.002	0.002		
$^7F_1$ Δ-Δ	0.001	0.001		
Binding energy	4.35	4.33	3.83	3.83

Partial-wave	Kappa			
	A	B	C	D
$^3D_2$ N-N	0.000	0.000	0.000	0.000
$^3D_2$ Δ-Δ	0.000	0.000		
$^7D_2$ Δ-Δ	0.000	0.000		
$^7G_2$ Δ-Δ	0.000	0.000		
Binding energy	-1.57	-1.57	-1.55	-1.55

Partial-wave	Kappa			
	A	B	C	D
$^3S_1$ N-N	0.021	0.022	0.019	0.020
$^3D_1$ N-N	0.057	0.072	0.059	0.071
$^3S_1$ Δ-Δ	0.003	0.002		
$^3D_1$ Δ-Δ	0.001	0.001		
$^7D_1$ Δ-Δ	0.015	0.016		
$^7G_1$ Δ-Δ	0.001	0.001		
Binding enery	-10.78	-11.70	-11.11	-11.78

Partial-wave	Kappa			
	A	B	C	D
$^3D_1$ N-N	0.000	0.000	0.000	0.000
$^3S_1$ N-N	0.000	0.001	0.000	0.001
$^3S_1$ Δ-Δ	0.000	0.000		
$^3D_1$ Δ-Δ	0.000	0.000		
$^7D_1$ Δ-Δ	0.000	0.000		
$^7G_1$ Δ-Δ	0.000	0.000		
Binding energy	0.53	0.59	0.53	0.58

Partial-wave	Kappa			
	A	B	C	D
$^1D_2$ N-N	0.000	0.000	0.000	0.000
$^5S_2$ N-Δ	0.000	0.000		
$^5D_2$ N-Δ	0.000	0.000		
$^5G_2$ N-Δ	0.000	0.000		
$^1D_2$ Δ-Δ	0.000	0.000		
$^5S_2$ Δ-Δ	0.000	0.000		
$^5D_2$ Δ-Δ	0.000	0.000		
$^5G_2$ Δ-Δ	0.000	0.000		
Binding energy	-0.82	-0.82	-0.87	-0.87

Partial-wave	Kappa			
	A	B	C	D
$^3P_2$ N-N	0.001	0.001	0.001	0.001
$^3F_2$ N-N	0.002	0.002	0.001	0.001
$^3P_2$ N-Δ	0.002	0.002		
$^3F_2$ N-Δ	0.001	0.001		
$^5P_2$ N-Δ	0.003	0.003		
$^5F_2$ N-Δ	0.004	0.004		
Binding energy	-2.45	-2.42	-2.48	-2.47

Partial-wave	Kappa			
	A	B	C	D
$^3F_2$ N-N	0.000	0.000	0.000	0.000
$^3P_2$ N-N	0.000	0.000	0.000	0.000
$^3P_2$ N-Δ	0.000	0.000		
$^3F_2$ N-Δ	0.000	0.000		
$^5P_2$ N-Δ	0.000	0.000		
$^5F_2$ N-Δ	0.000	0.000		
Binding energy	-0.16	-0.16	-0.16	-0.17

	A	B	C	D
Potential Energy	-23.22	-23.80	-26.41	-27.07
Kinetic Energy	16.74	16.74	16.74	16.74
Binding energy	-6.48	-7.06	-9.67	-10.33
Kappa	0.224	0.238	0.107	0.120
Resonance admixture	6.98	6.99		

Table 3. Results corresponding to the different sets of partial waves.

Fermi momentum: KF $= 1.16\ \mathrm{fm}^{-1}$
Density: RHO $= 0.11\ \mathrm{fm}^{-3}$
Average interparticle spacing: RM $= 1.31$ fm

Partial-wave	Kappa			
	A	B	C	D
${}^1S_0$ N-N	0.020	0.020	0.016	0.016
${}^5D_0$ N-Δ	0.048	0.044		
${}^1S_0$ Δ-Δ	0.003	0.002		
${}^5D_0$ Δ-Δ	0.007	0.008		
Binding energy	-9.74	-9.39	-11.49	-11.48

Partial-wave	Kappa			
	A	B	C	D
${}^1P_1$ N-N	0.003	0.003	0.002	0.002
${}^1P_1$ Δ-Δ	0.000	0.000		
${}^5P_1$ Δ-Δ	0.000	0.000		
${}^5F_1$ Δ-Δ	0.000	0.000		
Binding energy	1.06	1.06	1.10	1.10

Partial-wave	Kappa			
	A	B	C	D
${}^3P_0$ N-N	0.002	0.002	0.001	0.001
${}^3P_0$ N-Δ	0.001	0.001		
${}^3P_0$ Δ-Δ	0.000	0.000		
${}^7F_0$ Δ-Δ	0.001	0.001		
Binding energy	-2.07	-2.06	-2.16	-2.16

Partial-wave	Kappa			
	A	B	C	D
${}^3P_1$ N-N	0.006	0.006	0.003	0.003
${}^3P_1$ N-Δ	0.000	0.000		
${}^5P_1$ N-Δ	0.006	0.006		
${}^5F_1$ N-Δ	0.002	0.003		
${}^7F_1$ Δ-Δ	0.001	0.001		
Binding energy	6.24	6.20	5.43	5.43

Partial-wave	Kappa			
	A	B	C	D
${}^3D_2$ N-N	0.000	0.000	0.000	0.000
${}^3D_2$ Δ-Δ	0.000	0.000		
${}^7D_2$ Δ-Δ	0.000	0.000		
${}^7G_2$ Δ-Δ	0.000	0.000		
Binding energy	-2.32	-2.32	-2.29	-2.29

Partial-wave	Kappa			
	A	B	C	D
${}^3S_1$ N-N	0.025	0.025	0.021	0.022
${}^3D_1$ N-N	0.060	0.073	0.061	0.072
${}^3S_1$ Δ-Δ	0.003	0.003		
${}^3D_1$ Δ-Δ	0.001	0.001		
${}^7D_1$ Δ-Δ	0.017	0.019		
${}^7G_1$ Δ-Δ	0.001	0.001		
Binding enery	-12.19	-13.24	-12.58	-13.37

Partial-wave	Kappa			
	A	B	C	D
${}^3D_1$ N-N	0.000	0.000	0.000	0.000
${}^3S_1$ N-N	0.000	0.002	0.000	0.002
${}^3S_1$ Δ-Δ	0.000	0.000		
${}^3D_1$ Δ-Δ	0.000	0.000		
${}^7D_1$ Δ-Δ	0.000	0.000		
${}^7G_1$ Δ-Δ	0.000	0.000		
Binding energy	0.78	0.88	0.78	0.87

Partial-wave	Kappa			
	A	B	C	D
${}^1D_2$ N-N	0.000	0.000	0.000	0.000
${}^5S_2$ N-Δ	0.000	0.000		
${}^5D_2$ N-Δ	0.000	0.000		
${}^5G_2$ N-Δ	0.001	0.000		
${}^1D_2$ Δ-Δ	0.000	0.000		
${}^5S_2$ Δ-Δ	0.000	0.000		
${}^5D_2$ Δ-Δ	0.000	0.000		
${}^5G_2$ Δ-Δ	0.000	0.000		
Binding energy	-1.21	-1.21	-1.30	-1.30

Partial-wave	Kappa			
	A	B	C	D
${}^3P_2$ N-N	0.001	0.001	0.002	0.002
${}^3F_2$ N-N	0.002	0.002	0.002	0.001
${}^3P_2$ N-Δ	0.003	0.002		
${}^3F_2$ N-Δ	0.001	0.001		
${}^5P_2$ N-Δ	0.004	0.004		
${}^5F_2$ N-Δ	0.006	0.005		
Binding energy	-3.51	-3.46	-3.65	-3.62

Partial-wave	Kappa			
	A	B	C	D
${}^3F_2$ N-N	0.000	0.000	0.000	0.000
${}^3P_2$ N-N	0.000	0.000	0.000	0.000
${}^3P_2$ N-Δ	0.000	0.000		
${}^3F_2$ N-Δ	0.000	0.000		
${}^5P_2$ N-Δ	0.000	0.000		
${}^5F_2$ N-Δ	0.000	0.000		
Binding energy	-0.26	-0.26	-0.25	-0.25

Table 4. Results corresponding to the different sets of partial waves.

	A	B	C	D
Potential Energy	-26.07	-26.74	-30.60	-31.30
Kinetic Energy	19.75	19.75	19.75	19.75
Binding energy	-6.32	-6.99	-10.85	-11.56
Kappa	0.268	0.283	0.118	0.131
Resonance admixture	8.67	8.70		

Fermi momentum: KF $= 1.26\ \mathrm{fm}^{-1}$

Density: RHO $= 0.135\ \mathrm{fm}^{-3}$

Average interparticle spacing: RM $= 1.21$ fm

Partial-wave	Kappa			
	A	B	C	D
$^1S_0$ N-N	0.025	0.025	0.019	0.019
$^5D_0$ N-Δ	0.058	0.054		
$^1S_0$ Δ-Δ	0.003	0.003		
$^5D_0$ Δ-Δ	0.009	0.009		
Binding energy	-11.20	-10.77	-13.54	-13.53

Partial-wave	Kappa			
	A	B	C	D
$^1P_1$ N-N	0.004	0.004	0.003	0.003
$^1P_1$ Δ-Δ	0.000	0.000		
$^5P_1$ Δ-Δ	0.000	0.000		
$^5F_1$ Δ-Δ	0.000	0.000		
Binding energy	1.61	1.62	1.64	1.65

Partial-wave	Kappa			
	A	B	C	D
$^3P_0$ N-N	0.002	0.002	0.002	0.002
$^3P_0$ N-Δ	0.002	0.002		
$^3P_0$ Δ-Δ	0.000	0.000		
$^7F_0$ Δ-Δ	0.001	0.001		
Binding energy	-2.58	-2.58	-2.73	-2.73

Partial-wave	Kappa			
	A	B	C	D
$^3P_1$ N-N	0.008	0.008	0.003	0.003
$^3P_1$ N-Δ	0.000	0.000		
$^5P_1$ N-Δ	0.008	0.009		
$^5F_1$ N-Δ	0.003	0.004		
$^7F_1$ Δ-Δ	0.001	0.001		
Binding energy	8.69	8.61	7.45	7.46

Partial-wave	Kappa			
	A	B	C	D
$^3D_2$ N-N	0.000	0.000	0.000	0.000
$^3D_2$ Δ-Δ	0.000	0.000		
$^7D_2$ Δ-Δ	0.000	0.000		
$^7G_2$ Δ-Δ	0.000	0.000		
Binding energy	-3.28	-3.28	-3.24	-3.24

Partial-wave	Kappa			
	A	B	C	D
$^3S_1$ N-N	0.030	0.030	0.025	0.025
$^3D_1$ N-N	0.063	0.076	0.063	0.074
$^3S_1$ Δ-Δ	0.004	0.003		
$^3D_1$ Δ-Δ	0.001	0.001		
$^7D_1$ Δ-Δ	0.021	0.023		
$^7G_1$ Δ-Δ	0.001	0.002		
Binding enery	-13.45	-14.72	-13.88	-14.78

Partial-wave	Kappa			
	A	B	C	D
$^3D_1$ N-N	0.000	0.000	0.000	0.000
$^3S_1$ N-N	0.000	0.003	0.000	0.002
$^3S_1$ Δ-Δ	0.000	0.000		
$^3D_1$ Δ-Δ	0.000	0.000		
$^7D_1$ Δ-Δ	0.000	0.000		
$^7G_1$ Δ-Δ	0.000	0.000		
Binding energy	1.10	1.26	1.10	1.24

Partial-wave	Kappa			
	A	B	C	D
$^1D_2$ N-N	0.000	0.000	0.000	0.000
$^5S_2$ N-Δ	0.001	0.001		
$^5D_2$ N-Δ	0.001	0.001		
$^5G_2$ N-Δ	0.001	0.001		
$^1D_2$ Δ-Δ	0.000	0.000		
$^5S_2$ Δ-Δ	0.000	0.000		
$^5D_2$ Δ-Δ	0.000	0.000		
$^5G_2$ Δ-Δ	0.000	0.000		
Binding energy	-1.72	-1.72	-1.87	-1.87

Partial-wave	Kappa			
	A	B	C	D
$^3P_2$ N-N	0.001	0.001	0.002	0.002
$^3F_2$ N-N	0.003	0.002	0.002	0.001
$^3P_2$ N-Δ	0.003	0.003		
$^3F_2$ N-Δ	0.001	0.001		
$^5P_2$ N-Δ	0.005	0.005		
$^5F_2$ N-Δ	0.008	0.008		
Binding energy	-4.84	-4.77	-5.15	-5.11

Partial-wave	Kappa			
	A	B	C	D
$^3F_2$ N-N	0.000	0.000	0.000	0.000
$^3P_2$ N-N	0.000	0.000	0.000	0.000
$^3P_2$ N-Δ	0.000	0.000		
$^3F_2$ N-Δ	0.000	0.000		
$^5P_2$ N-Δ	0.000	0.000		
$^5F_2$ N-Δ	0.000	0.000		
Binding energy	-0.41	-0.40	-0.39	-0.39

Table 5. Results corresponding to the different sets of partial waves.

	A	B	C	D
Potential Energy	-28.56	-29.27	-34.67	-35.35
Kinetic Energy	23.01	23.01	23.01	23.01
Binding energy	-5.55	-6.26	-11.65	-12.34
Kappa	0.322	0.338	0.132	0.144
Resonance admixture	10.71	10.77		

Fermi momentum: KF $= 1.36\ \mathrm{fm}^{-1}$

Density: RHO $= 0.170\ \mathrm{fm}^{-3}$

Average interparticle spacing: RM $= 1.12$ fm

Partial-wave	Kappa			
	A	B	C	D
$^1S_0$ N-N	0.031	0.031	0.022	0.022
$^5D_0$ N-Δ	0.070	0.065		
$^1S_0$ Δ-Δ	0.004	0.003		
$^5D_0$ Δ-Δ	0.010	0.011		
Binding energy	-12.64	-12.14	-15.62	-15.62

Partial-wave	Kappa			
	A	B	C	D
$^1P_1$ N-N	0.005	0.005	0.004	0.004
$^1P_1$ Δ-Δ	0.000	0.000		
$^5P_1$ Δ-Δ	0.001	0.001		
$^5F_1$ Δ-Δ	0.001	0.001		
Binding energy	2.38	2.39	2.40	2.41

Partial-wave	Kappa			
	A	B	C	D
$^3P_0$ N-N	0.003	0.003	0.002	0.002
$^3P_0$ N-Δ	0.002	0.002		
$^3P_0$ Δ-Δ	0.000	0.000		
$^7F_0$ Δ-Δ	0.001	0.001		
Binding energy	-3.12	-3.11	-3.34	-3.34

Partial-wave	Kappa			
	A	B	C	D
$^3P_1$ N-N	0.011	0.010	0.004	0.004
$^3P_1$ N-Δ	0.000	0.001		
$^5P_1$ N-Δ	0.011	0.012		
$^5F_1$ N-Δ	0.004	0.005		
$^7F_1$ Δ-Δ	0.002	0.001		
Binding energy	11.77	11.63	9.96	9.98

Partial-wave	Kappa			
	A	B	C	D
$^3D_2$ N-N	0.000	0.000	0.000	0.000
$^3D_2$ Δ-Δ	0.000	0.000		
$^7D_2$ Δ-Δ	0.000	0.000		
$^7G_2$ Δ-Δ	0.000	0.000		
Binding energy	-4.46	-4.46	-4.41	-4.41

Partial-wave	Kappa			
	A	B	C	D
$^3S_1$ N-N	0.036	0.036	0.030	0.030
$^3D_1$ N-N	0.067	0.079	0.066	0.075
$^3S_1$ Δ-Δ	0.004	0.003		
$^3D_1$ Δ-Δ	0.001	0.001		
$^7D_1$ Δ-Δ	0.024	0.027		
$^7G_1$ Δ-Δ	0.001	0.002		
Binding enery	-14.55	-16.01	-14.94	-15.92

Partial-wave	Kappa			
	A	B	C	D
$^3D_1$ N-N	0.000	0.000	0.000	0.000
$^3S_1$ N-N	0.000	0.005	0.000	0.004
$^3S_1$ Δ-Δ	0.000	0.000		
$^3D_1$ Δ-Δ	0.000	0.000		
$^7D_1$ Δ-Δ	0.000	0.000		
$^7G_1$ Δ-Δ	0.000	0.000		
Binding energy	1.49	1.74	1.49	1.70

Partial-wave	Kappa			
	A	B	C	D
$^1D_2$ N-N	0.000	0.000	0.000	0.000
$^5S_2$ N-Δ	0.001	0.001		
$^5D_2$ N-Δ	0.001	0.001		
$^5G_2$ N-Δ	0.001	0.001		
$^1D_2$ Δ-Δ	0.000	0.000		
$^5S_2$ Δ-Δ	0.000	0.000		
$^5D_2$ Δ-Δ	0.000	0.000		
$^5G_2$ Δ-Δ	0.000	0.000		
Binding energy	-2.36	-2.36	-2.60	-2.60

Partial-wave	Kappa			
	A	B	C	D
$^3P_2$ N-N	0.001	0.001	0.002	0.002
$^3F_2$ N-N	0.003	0.003	0.002	0.002
$^3P_2$ N-Δ	0.005	0.005		
$^3F_2$ N-Δ	0.001	0.002		
$^5P_2$ N-Δ	0.007	0.007		
$^5F_2$ N-Δ	0.011	0.01		
Binding energy	-6.47	-6.37	-7.04	-6.98

Partial-wave	Kappa			
	A	B	C	D
$^3F_2$ N-N	0.000	0.000	0.000	0.000
$^3P_2$ N-N	0.000	0.000	0.000	0.000
$^3P_2$ N-Δ	0.000	0.000		
$^3F_2$ N-Δ	0.000	0.000		
$^5P_2$ N-Δ	0.000	0.000		
$^5F_2$ N-Δ	0.000	0.000		
Binding energy	-0.60	-0.59	-0.57	-0.56

	A	B	C	D
Potential Energy	-30.42	-31.05	-38.44	-39.02
Kinetic Energy	26.52	26.52	26.52	26.52
Binding energy	-3.90	-4.53	-11.92	-12.51
Kappa	0.385	0.405	0.147	0.160
Resonance admixture	13.15	13.24		

Table 6. Results corresponding to the different sets of partial waves.

Fermi momentum: KF = 1.46 fm^{-1}
Density: RHO = 0.210 fm^{-3}
Average interparticle spacing: RM = 1.04 fm

Partial-wave	Kappa			
	A	B	C	D
$^1S_0$ N-N	0.038	0.038	0.026	0.026
$^5D_0$ N-Δ	0.083	0.077		
$^1S_0$ Δ-Δ	0.005	0.004		
$^5D_0$ Δ-Δ	0.012	0.013		
Binding energy	-14.04	-13.44	-17.69	-17.69

Partial-wave	Kappa			
	A	B	C	D
$^1P_1$ N-N	0.006	0.006	0.005	0.005
$^1P_1$ Δ-Δ	0.000	0.000		
$^5P_1$ Δ-Δ	0.001	0.001		
$^5F_1$ Δ-Δ	0.001	0.001		
Binding energy	3.42	3.43	3.40	3.42

Partial-wave	Kappa			
	A	B	C	D
$^3P_0$ N-N	0.005	0.005	0.003	0.003
$^3P_0$ N-Δ	0.003	0.003		
$^3P_0$ Δ-Δ	0.000	0.000		
$^7F_0$ Δ-Δ	0.002	0.002		
Binding energy	-3.65	-3.64	-3.96	-3.96

Partial-wave	Kappa			
	A	B	C	D
$^3P_1$ N-N	0.014	0.014	0.005	0.004
$^3P_1$ N-Δ	0.001	0.001		
$^5P_1$ N-Δ	0.015	0.017		
$^5F_1$ N-Δ	0.006	0.007		
$^7F_1$ Δ-Δ	0.002	0.002		
Binding energy	15.60	15.37	13.02	13.04

Partial-wave	Kappa			
	A	B	C	D
$^3D_2$ N-N	0.001	0.001	0.000	0.000
$^3D_2$ Δ-Δ	0.000	0.000		
$^7D_2$ Δ-Δ	0.000	0.000		
$^7G_2$ Δ-Δ	0.000	0.000		
Binding energy	-5.88	-5.88	-5.83	-5.83

Partial-wave	Kappa			
	A	B	C	D
$^3S_1$ N-N	0.043	0.043	0.036	0.036
$^3D_1$ N-N	0.071	0.082	0.068	0.076
$^3S_1$ Δ-Δ	0.005	0.004		
$^3D_1$ Δ-Δ	0.002	0.001		
$^7D_1$ Δ-Δ	0.029	0.031		
$^7G_1$ Δ-Δ	0.001	0.002		
Binding enery	-15.39	-16.99	-15.67	-16.71

Partial-wave	Kappa			
	A	B	C	D
$^3D_1$ N-N	0.000	0.000	0.000	0.000
$^3S_1$ N-N	0.000	0.009	0.000	0.005
$^3S_1$ Δ-Δ	0.000	0.000		
$^3D_1$ Δ-Δ	0.000	0.000		
$^7D_1$ Δ-Δ	0.000	0.000		
$^7G_1$ Δ-Δ	0.000	0.000		
Binding energy	1.95	2.35	1.95	2.27

Partial-wave	Kappa			
	A	B	C	D
$^1D_2$ N-N	0.000	0.000	0.000	0.000
$^5S_2$ N-Δ	0.002	0.001		
$^5D_2$ N-Δ	0.001	0.001		
$^5G_2$ N-Δ	0.002	0.002		
$^1D_2$ Δ-Δ	0.000	0.000		
$^5S_2$ Δ-Δ	0.000	0.000		
$^5D_2$ Δ-Δ	0.000	0.000		
$^5G_2$ Δ-Δ	0.000	0.000		
Binding energy	-3.15	-3.14	-3.51	-3.51

Partial-wave	Kappa			
	A	B	C	D
$^3P_2$ N-N	0.001	0.001	0.002	0.002
$^3F_2$ N-N	0.003	0.003	0.002	0.002
$^3P_2$ N-Δ	0.006	0.006		
$^3F_2$ N-Δ	0.002	0.002		
$^5P_2$ N-Δ	0.010	0.010		
$^5F_2$ N-Δ	0.014	0.014		
Binding energy	-8.44	-8.28	-9.34	-9.26

Partial-wave	Kappa			
	A	B	C	D
$^3F_2$ N-N	0.000	0.000	0.000	0.000
$^3P_2$ N-N	0.000	0.000	0.000	0.000
$^3P_2$ N-Δ	0.000	0.000		
$^3F_2$ N-Δ	0.000	0.000		
$^5P_2$ N-Δ	0.000	0.000		
$^5F_2$ N-Δ	0.000	0.000		
Binding energy	-0.85	-0.83	-0.80	-0.79

Table 7. Results corresponding to the different sets of Partial waves.

	A	B	C	D
Potential Energy	-31.85	-32.07	-41.72	-42.12
Kinetic Energy	30.28	30.28	30.28	30.28
Binding energy	-1.57	-1.79	-11.45	-11.84
Kappa	0.462	0.488	0.165	0.180
Resonance admixture	16.14	16.24		

Fermi momentum: KF = 1.56 fm^{-1}
Density: RHO = 0.256 fm^{-3}
Average interparticle spacing: RM = 0.98 fm

Partial-wave	Kappa			
	A	B	C	D
$^1S_0$ N-N	0.045	0.045	0.031	0.031
$^5D_0$ N-Δ	0.039	0.092		
$^1S_0$ Δ-Δ	0.005	0.005		
$^5D_0$ Δ-Δ	0.014	0.015		
Binding energy	-15.49	-14.74	-19.68	-19.70

Partial-wave	Kappa			
	A	B	C	D
$^1P_1$ N-N	0.007	0.007	0.006	0.005
$^1P_1$ Δ-Δ	0.000	0.000		
$^5P_1$ Δ-Δ	0.001	0.001		
$^5F_1$ Δ-Δ	0.001	0.001		
Binding energy	4.76	4.78	4.71	4.74

Partial-wave	Kappa			
	A	B	C	D
$^3P_0$ N-N	0.006	0.006	0.004	0.004
$^3P_0$ N-Δ	0.004	0.004		
$^3P_0$ Δ-Δ	0.000	0.000		
$^7F_0$ Δ-Δ	0.002	0.002		
Binding energy	-4.14	-4.12	-4.56	-4.56

Partial-wave	Kappa			
	A	B	C	D
$^3P_1$ N-N	0.018	0.017	0.005	0.005
$^3P_1$ N-Δ	0.001	0.002		
$^5P_1$ N-Δ	0.020	0.023		
$^5F_1$ N-Δ	0.008	0.010		
$^7F_1$ Δ-Δ	0.003	0.002		
Binding energy	20.22	19.87	16.68	16.70

Partial-wave	Kappa			
	A	B	C	D
$^3D_2$ N-N	0.001	0.001	0.000	0.000
$^3D_2$ Δ-Δ	0.000	0.000		
$^7D_2$ Δ-Δ	0.000	0.000		
$^7G_2$ Δ-Δ	0.000	0.001		
Binding energy	-7.56	-7.56	-7.51	-7.51

Partial-wave	Kappa			
	A	B	C	D
$^3S_1$ N-N	0.052	0.052	0.043	0.042
$^3D_1$ N-N	0.076	0.086	0.071	0.078
$^3S_1$ Δ-Δ	0.006	0.005		
$^3D_1$ Δ-Δ	0.002	0.001		
$^7D_1$ Δ-Δ	0.034	0.036		
$^7G_1$ Δ-Δ	0.002	0.002		
Binding enery	-16.06	-17.64	-16.01	-17.05

Partial-wave	Kappa			
	A	B	C	D
$^3D_1$ N-N	0.000	0.000	0.071	0.078
$^3S_1$ N-N	0.000	0.016	0.000	0.000
$^3S_1$ Δ-Δ	0.000	0.000		
$^3D_1$ Δ-Δ	0.000	0.000		
$^7D_1$ Δ-Δ	0.000	0.000		
$^7G_1$ Δ-Δ	0.000	0.000		
Binding energy	2.48	3.12	2.49	2.96

Partial-wave	Kappa			
	A	B	C	D
$^1D_2$ N-N	0.000	0.000	0.000	0.000
$^5S_2$ N-Δ	0.002	0.002		
$^5D_2$ N-Δ	0.002	0.002		
$^5G_2$ N-Δ	0.002	0.002		
$^1D_2$ Δ-Δ	0.000	0.000		
$^5S_2$ Δ-Δ	0.000	0.000		
$^5D_2$ Δ-Δ	0.000	0.000		
$^5G_2$ Δ-Δ	0.000	0.000		
Binding energy	-4.10	-4.09	-4.63	-4.63

Partial-wave	Kappa			
	A	B	C	D
$^3P_2$ N-N	0.002	0.002	0.002	0.002
$^3F_2$ N-N	0.004	0.004	0.003	0.002
$^3P_2$ N-Δ	0.008	0.008		
$^3F_2$ N-Δ	0.003	0.003		
$^5P_2$ N-Δ	0.013	0.013		
$^5F_2$ N-Δ	0.019	0.018		
Binding energy	-10.80	-10.56	-12.11	-11.99

Partial-wave	Kappa			
	A	B	C	D
$^3F_2$ N-N	0.000	0.000	0.000	0.000
$^3P_2$ N-N	0.000	0.001	0.000	0.000
$^3P_2$ N-Δ	0.000	0.000		
$^3F_2$ N-Δ	0.000	0.000		
$^5P_2$ N-Δ	0.000	0.000		
$^5F_2$ N-Δ	0.000	0.000		
Binding energy	-1.16	-1.12	-1.10	-1.08

Table 8.

	a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4
A	– 36.85	127.61	– 186.5	56.15	25.67	– 66.31	61.14	– 16.29
C	– 58.80	172.69	– 206.4	49.64	12.21	– 31.27	31.41	– 6.86

it is possible to deduce a one- and a two particle equation concerning the ground state $|\Psi\rangle$ which contains the resonance-configurations explicitly.

Specializing to nuclear matter the one-particle equation is trivial while the two-particle equation turns out to be a generalized Bethe-Goldstone-equation.

To solve the systems of integro-differential equations which represent the two-particle equation, the interactions containing resonances are described by potentials, which we have calculated using the static limit of the meson theory. The interactions involving only nucleons on the other hand are described with the help of the phenomenological Reid-potential. Of course the Reid potential was renormalized first in order to reproduce the experimental two particle dates (scattering phase-shifts; binding energy of the deuteron), although the resonances were treated explicitly. The numerical results, we want to discuss now are given in detail in the Tables 1—7.

A nuclear matter calculation which takes only nucleonic-configurations into consideration shows saturation properties as represented in Figure 3. If the resonance configurations are treated explicitly we get the saturation properties shown in Figure 3. Comparison shows that the explicit treatment of the resonances moves the equilibrium point considerably — the density as well as the corresponding binding energy. Treating only nucleonic-configurations we calculate a maximum binding energy per nucleon of 12 MeV and a corresponding density of 0.21 fm^{-3}, while a calculation treating the resonances explicitly amounts to 7 MeV per nucleon at 0.117 fm^{-3}. The "experimental" equilibrium point is determined by 16 MeV per nucleon and 0.170 fm^{-3}.

Although treating the resonances explicitly expands the configuration space considerably the calculated equilibrium point does not move towards the experimental point — as one might hope perhaps — but moves away considerably. Another effect worth mentioning is the fact that the explicit treatment of the resonances nearly doubles the value of the defect integral $\varkappa$, as shown in Fig. 5

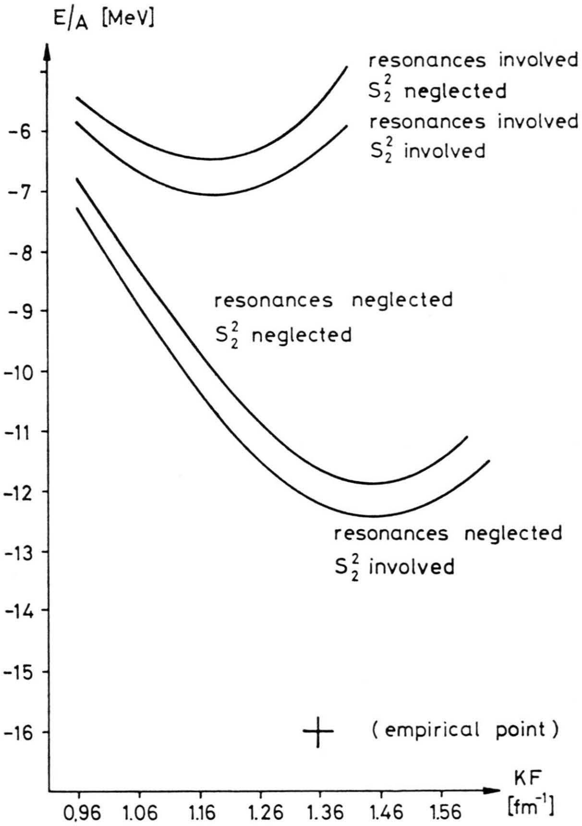

Fig. 3. Saturation properties in nuclear matter.

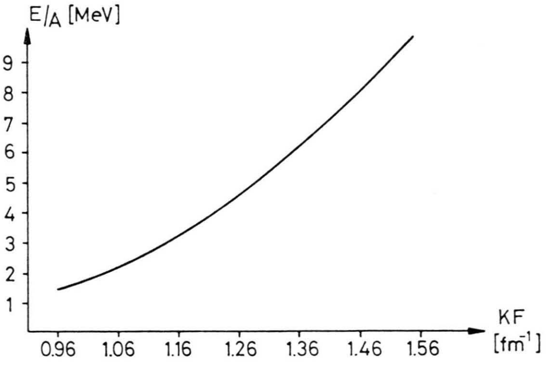

Fig. 4. Loss of binding energy due to resonances.

and 6. The defect integral $\varkappa$ tells something about the energy contribution due to three-particle correlations compared to the energy contribution due to two-particle correlations, see Reference [12]. Because of $\varkappa$ we should expect an energy contribution of 5 to 6 MeV due to three particle correlations instead of 2 to 3 MeV as estimated by some authors [6, 13]. Strongly correlated to the magnitude of the defect integrals is the resonance admixture as shown in Figure 7.

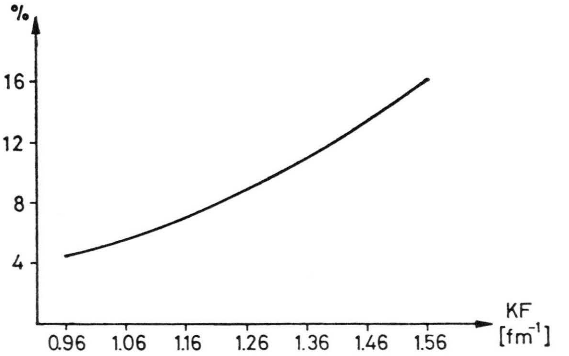

Fig. 5. Defect integrals (resonances neglected).

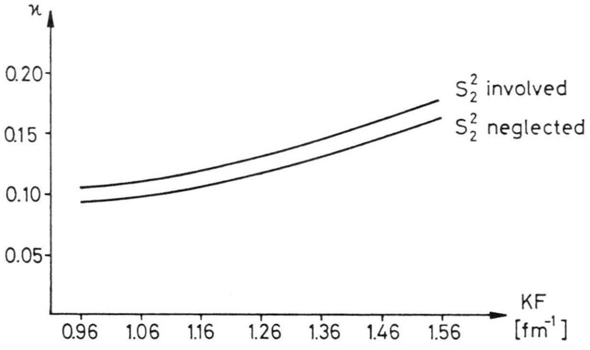

Fig. 6. Defect integrals (resonances included).

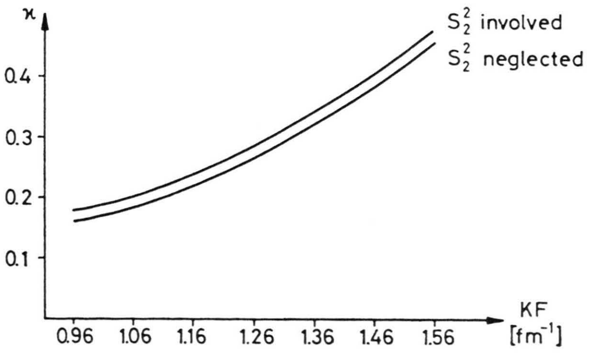

Fig. 7. Resonance admixture.

A calculation done for the saturation density $\varrho = 0.117\ \mathrm{fm}^{-3}$ produces a resonance admixture of nearly 8%. It is worth mentioning that the delta-delta-components of the ground state wavefunction contribution to this value is nearly the same as that of the nucleon delta components, to which Day and Coester did restrict [14].

Alltogether we can say that the explicit treatment of the resonance in nuclear matter shows significant effects. To draw conclusions concerning the two particle potentials we have used we have to ensure at first that the calculated effects are not reduced due to an unreasonable application of many body theory nor to a faulty numerical calculation of the two particle equation. For the following reasons we believe the probability for this to be negligible. First of all doing the numerical calculation of the two particle equation we have used only controllable approximations. A look at the extensive description of our numeric in Chapter 9 shows that we are sure to solve Eq. (6.15) indeed and not something else. Next our program is able to compute the known nuclear-matter (all resonance neglected) as a special case.

The results agree with earlier calculations, a hint that our program works correctly. At last our numerical results become credible by the fact that comparable calculations based on the Bethe-Goldstone equation produce similar results. Green and Niskanen [15] conclude that the explicit treatment of the Δ-resonance produces an equilibrium point $\varrho = 0.117\ \mathrm{fm}^{-3}$ and $E_{\mathrm{pot}} = 8$ MeV. Day and Coester [14] calculate a binding energy $E_{\mathrm{bin}} = 5$ MeV corresponding to a density $\varrho = 0.170\ \mathrm{fm}^{-3}$.

For the following reasons one should not deduce the remarkable resonance effects to a faulty description of the resonances by the many body theory. We have mentioned already that we treat the resonances like excited nucleon states. Our description of the resonances in nuclear matter is consistent to that in the two particle system [16, 17]. This is important, because the Reid-potential is fitted to reproduce the experimental data (scattering phase shifts, binding energy of the deuteron) of the two particle system.

There are two problems however, which are typical for a many body calculation and which are worth thinking about. On the one hand one has to check if it is reasonable to neglect three- and more particle effects. On the other hand one has to think about the correct way to treat the important Pauli projection operators. Both problems don't exist in the two-particle problem. Concerning the three- and more particle effects we already said something above. The contribution to the binding energy due to three- and four particle effects are not known yet. The estimates concerning these effects differ considerably. Probable — hopefully — these effects are small. Moreover we don't see a reason why three- and four particle effects should be more important if one treats the delta resonances explicitly. Thus we are convinced that our restriction to two-particle effects is justified. Concerning the correct treatment of the Pauli projection operators when resonances are involved we see a problem which is somewhat more complicated. If two nucleons scatter in the Fermi sea there are only two possibilities for the final state because of the Pauli principle. Either the nucleons scatter back to their original one particle states or they leave the Fermi sea. The Pauli principle therefore limits the possible interactions very strongly. The picture looks different if at least one of the nucleons changes it's internal structure — becomes a resonance — caused by the interaction. The spin as well as the isospin quantum numbers of the resonances are different from that of the nucleon. So the scattering of the delta is not restricted by the Pauli principle and it may scatter to a point in the fermi sea. The Pauli principle in this form favours the delta production. If on the contrary one looks at the deltas as a resonant nucleon-pion state, the Pauli principle should forbid scattering in the Fermi sea with respect to the nucleon involved in the resonance state. In this way the resonance production is not favoured. We treated the Pauli principle like Day and Coester [14].

Treating the Pauli principle in our way is consistent with the concept of the delta as an excited nucleon. Treating the Pauli principle the other way one should consistently solve a three particle equation — two particles being nucleons and the third being a pion. We conclude that the remarkable effects we get by treating the delta resonances explicitly should not be traced to a faulty numerical calculation nor to an inadequate treatment of the deltas by many body theory. The question is now which shortcoming of our two body potentials may be responsible for the discrepancy between theory and experiment.

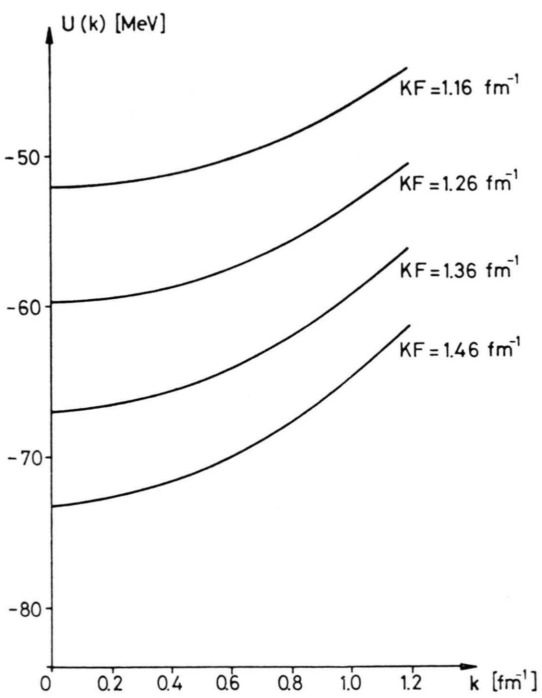

Fig. 8. One particle potentials (resonances neglected).

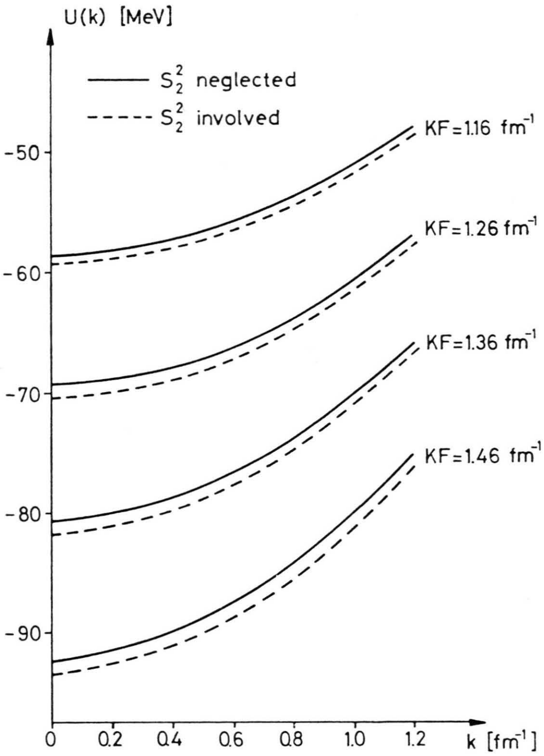

Fig. 9. One particle potentials (resonances neglected).

The two-particle-potential we have used can be devided into two parts. The nucleon-nucleon interaction is described by a modified Reid-potential. The interactions which involve resonances are described by meson theoretical potentials.

To use the Reid potential in nuclear physics is meaningful only because this potential permits the reproduction of the experimental two particle data. Only the long range part of the Reid-potential is determined by the one pion exchange. The fact, that there exist lots of "phase shift equivalent" potentials shows however that a potential is not uniquely determined by the experimental two particle dates. The remarkable shift of the saturation point caused by the explicit treatment of the resonances should be a hint that the two particle interaction is determined by a lot of competing effects which have to be studied carefully if one wants to describe the two particle interaction in a realistic way. The discrepancy between theory and experiment which is enlarged by our calculation should not cause the rejection of our resonance potentials but should intensify the demand for diminishing the phenomenologically part of the two body interaction.

On the other hand we do not want to conceal that the theoretical description of the resonance potentials should be improved. Calculating resonance potentials one is confronted to some critical points. For an overall discussion see Reference [11].

The remarkable effects concerning the saturation properties of nuclear matter we calculated by treating the resonances explicitly show that it is very important for nuclear calculations to treat the baryonic degrees of freedom as precisely as possible.

A better understanding of the two particle interaction can result only from a mesonic treatment of the interaction. To extend this treatment will be the task for the near future. In this context a nuclear matter calculation proves to be a very sensible instrument for testing the two particle potentials.

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